This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

**Usage guidelines**

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

+ *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.

+ *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.

+ *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.

+ *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

**About Google Book Search**

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at `http://books.google.com/`

OF CALIFORNIA
TMENT OF CIVIL ENGINEERING
BERKELEY, CALIFORNIA

Engineering
Library

DEPARTMENT OF CIVIL ENGINEERING
BERKELEY, CALIFORNIA

# REINFORCED CONCRETE DESIGN

## VOL. II. PRACTICE

BY

OSCAR FABER

O.B.E., D.Sc., A.M.Inst.C.E., A.C.G.I., etc.

CONSULTING ENGINEER
CHIEF ENGINEER TO TROLLOPE AND COLLS, LONDON
CONSULTING ENGINEER TO H.M. OFFICE OF WORKS
CIVIL ENGINEER TO THE CALICO PRINTERS ASSOCIATION
HEAD OF ENGINEERING SIDE OF ARCHITECTURAL ASSOCIATION SCHOOL
COUNCIL MEMBER CONCRETE INSTITUTE

NEW YORK
LONGMANS, GREEN & CO.
LONDON: EDWARD ARNOLD
1920

*All rights reserved*

TA683
F2
1919
V.2
Engineering
Library

Civil Eng Dept

UNIV. OF
CALIFORNIA

# PREFACE

THE reception of Volume I of *Reinforced Concrete Design* has been very gratifying, particularly as appreciation seems to increase rather as the years roll on, possibly as the subject matter is more universally known and used.

Parts of Volume I the author does not feel he can add to with great advantage, particularly the portion dealing with the resistance of beams, slabs, and columns to bending, compression and tension, and combinations of these actions.

When it comes to the determination of bending moments in beams and columns, the mathematical treatment and resultant formulae are given in Volume I, but the author has found that few designers have the time and ability to apply these formulae as presented. He has also found that while designers have little difficulty in calculating resistances of members to known straining actions, using the treatment of Volume I, and will therefore be meticulously accurate not to exceed safe stresses by a few per cent., yet their inaccuracies in estimating the moments and other straining actions, where errors of fifty and even a hundred per cent. are not uncommon, throw this accuracy to the winds, and make it quite misleading and fictitious.

These considerations made it clear to the author that in his second volume he must deal more fully and principally with the ready determination of correct moments in beams and columns, and he has no apology to offer for the fact that this second volume so largely concerns itself with these matters.

It has, however, been his aim, not only to secure much greater *accuracy* in this important matter, but also to effect for the designer an *important saving of time* in his everyday practice. It is for this reason that Volume II has been named *practice* and Volume I

449483

*theory*, in that, while much of the basis of Volume II either exists or can be derived from Volume I, it is in Volume II given in a form convenient for ready reference.

To this end, actual bending moment diagrams for beams are given for any number of spans, any ratios of live load to dead, any conditions of loading, any fixity with monolithic columns of different relative stiffnesses, and in any practical case, the designer will find the appropriate curve, from which he has simply to read off the maximum positive and negative moment to provide against at any distance along the span.

No apology is needed for the very simple curves from which the moments in columns can be derived by simple inspection. At last the importance of these moments is coming to be recognized and people who ignore them to-day are guilty of criminal negligence.

With the object of still further reducing arithmetical and mathematical labour to a minimum, useful series of standard beams and columns are given with all their properties fully evaluated. From these, the calculation of the important quantity $\frac{KC}{B}$ is very readily obtained, and on this so much depends. The use of standard beams and columns also has many obvious practical advantages.

The author, since the publication of Volume I, has made many researches on the question of shearing resistance of concrete beams, which have resulted in an understanding of this subject so much more complete that a chapter is devoted to a résumé of the results, and the fuller treatment can be referred to if desired.

The moment curves given in Appendix II of continuous beams, with the stiffness of columns taken into account, are considered by the author to make this important subject within the scope of practical usefulness for the first time, and to enable a very important element of strength and economy to be given its proper value by inspection.

The treatment of unequal spans is also quite original, and, it is hoped, enables these quite common cases to be dealt with almost by inspection, whereas hitherto very grievous errors have been incurred through the complete inadequacy of any known treatment not too onerous for practical use.

OSCAR FABER.

VALHALLA,
FARTHING DOWN,
COULSDON, SURREY.

*August*, 1920.

# CONTENTS


| | | PAGE |
|---|---|---|
| PREFACE | | iii |
| CONTENTS | | vii |
| LIST OF SYMBOLS | | x |

## PART I

### BENDING MOMENTS IN CONTINUOUS BEAMS NEGLECTING STIFFNESS OF COLUMNS

| | | |
|---|---|---|
| CHAP. I. | PRELIMINARY<br>Live Load and Dead Load<br>Parabolic Ordinates and Construction<br>Maximum Moment with Combined Loadings | 1 |
| CHAP. II. | TWO SPANS, UNIFORM LOADING | 9 |
| CHAP. III. | TWO SPANS, CENTRAL POINT LOADING | 16 |
| CHAP. IV. | TWO SPANS, THIRD POINT LOADINGS | 28 |
| CHAP. V. | MANY SPANS, UNIFORM LOADING | 40 |
| CHAP. VI. | MANY SPANS, CENTRAL POINT LOADING | 58 |
| CHAP. VII. | MANY SPANS, THIRD POINT LOADINGS | 68 |
| CHAP. VIII. | OTHER ARRANGEMENTS OF LOADING<br>Theorem of Three Moments | 78 |
| CHAP. IX. | ROLLING LOADS | 90 |
| CHAP. X. | UNEQUAL SPANS | 94 |

## PART II

### MOMENTS IN COLUMNS

PAGE

CHAP. XI. OUTSIDE COLUMNS . . . . . 123

CHAP. XII. INTERIOR COLUMNS . . . . . 139

## PART III

CHAP. XIII. STANDARD COLUMNS AND T-BEAMS AND THEIR PROPERTIES CALCULATED . . . . . 147

CHAP. XIV. LIVE LOAD ALLOWANCE FOR ROLLING LOADS . . 159

CHAP. XV. SHEAR RESISTANCE OF R.C. BEAMS . . . . . 163

APPENDIX I. ALIGNMENT DIAGRAMS FOR R.C. BEAMS . . . 172

APPENDIX II. BENDING MOMENTS IN CONTINUOUS BEAMS, ALLOWING FOR STIFFNESS OF MONOLITHIC COLUMNS 175

One span, uniform loading, Fig. 73 . . . 175

„ central point loading, Fig. 83

„ third point loading, Fig. 83

Two spans, uniform loading, Fig. 77 . . . 176

„ central point loading, Fig. 84

„ third point loading, Fig. 85

Many spans, uniform loading, dead load, Fig. 78 185

„ „ „ live load, Fig. 82

„ central point loading, dead, Fig. 86

„ „ „ live, Fig. 87

„ third point loading, dead, Fig. 88

„ „ „ live, Fig. 89

APPENDIX III. L.C.C. REGULATIONS FOR REINFORCED CONCRETE . 205

Part I. General . . . . . 205

Part II. Data to be used . . . . . 207

*Loads, Wind Pressure, Depth to Span, Bending Moments, Working Stresses, Adhesion, Modular Ratios*

PAGE

Part III. Beams . . . . . 218
*Compressive Reinforcement*
*Shear Reinforcement*
*Splays*
*Slabs*
*Resistance Moments*

Part IV. Pillars and other Struts . . . . . 228

Part V. Walls . . . . . 234

Part VI. Foundations . . . . . 236

Part VII. Protection . . . . . 237

Part VIII. Materials and Testing . . . . . 237

Part IX. Formwork or Centering . . . . . 242

Part X. Workmanship . . . . . 242

AUTHOR'S NOTES ON L.C.C. REGULATIONS . . . . . 244

INDEX . . . . . 245

# LIST OF SYMBOLS

## LENGTHS, DISTANCES, INTENSITY OF LOADS, STRESSES PER UNIT OF AREA, AND CONSTANTS

$a$ Arm of the couple formed by the compressive and tensile forces in a beam.
$a_1$ Ratio $a/d$.
$b$ Breadth of a rectangular beam.
$b_r$ Breadth of the rib in a T-beam.
$b_s$ Effective breadth of the slab in a T-beam.
$c$ Compressive stress intensity.
c. Compressive stress intensity on concrete.
$c_s$ Compressive stress intensity on steel.
$d$ Effective depth of a beam from top of beam to axis of tensile reinforcements.
$d_t$ Total depth.
$d_s$ Total depth of a slab in a T-beam.
$d_c$ Depth or distance of the centre of compressive reinforcement from the compressed edge.
$\delta$ Deflection of a beam.
$e$ Eccentricity of any load.
$f$ Friction or adhesion between surfaces in units of force per unit of area.
$h$ Height.
$l$ Length.
$l$ Effective length or span of a beam or arch.
$m$ Modular ratio, *i.e.* the ratio between the elastic moduli of steel and concrete $= E_s/E_c$.
$n$ In beams: distance of the neutral axis from the compressed edge of a beam.
$n_1$ The ratio $n/d$, *i.e.* the distance between the neutral axis and the compressed edge divided by the effective depth of a beam.
$p$ Percentage of steel, *i.e.* $p = 100r$.
$p$ Intensity of pressure per unit of length or area in any direction.
$r$ Radius.
$r$ Ratio of area of steel to area of concrete in single reinforced beams (compare $p$).
$t$ Tensile stress intensity.
$t_1$ Ratio of stresses $= t/c$.
$w$ Weight per unit of length.
$w$ Weight per unit of volume.
$x$ Horizontal co-ordinate of a point.
$y$ Vertical co-ordinate of a point.

## AREAS, VOLUMES, MOMENTS, TOTAL LOADS, TOTAL FORCES, AND CONSTANTS

A Total cross-sectional area of a pillar.
$A_l$ Cross-sectional area of longitudinal steel rods in a pillar.
$A_e$ Equivalent area.
$A_c$ Area of compressive reinforcement in beams.
$A_t$ Area of tensile reinforcement in beams.
B Ratio $I/l$ for beams, *i.e.* a measure of stiffness.
C Ratio $I/l$ for columns, *i.e.* a measure of stiffness.
E Elastic modulus of any material.
$E_c$ Elastic modulus of concrete in compression.
$E_s$ Elastic modulus of steel.
F Total friction between any two surfaces.
I Inertia moment.
K A constant in the equation $M = KCE\alpha$ (see Appendix II. (1)).
M Bending moment.
P Total pressure on a given area.
R Resistance moment of the internal stresses in a beam at a given cross-section.
R Reaction of a beam on its support.
S Total shearing force across a section.
T Total tensile force.
W Weight or load.
$\frac{KC}{B}$ An expression for relative stiffness of beams to columns. For values of K see Vol. I, p. 125. C is moment of inertia of column divided by its length, B the same for the beam.

## ANGLES, CONSTANTS, AND MISCELLANEOUS

$\alpha$ Slope of a beam or column at the end produced by bending.
$\theta$ An angle.
$\mu$ Coefficient of friction.

FOLDING PLATE

Alignment Diagram for Rectangular Reinforced Concrete Beams

TO FACE PAGE 174

UNIV. OF CALIFORNIA

# PART I

## BENDING MOMENTS IN CONTINUOUS BEAMS NEGLECTING STIFFNESS OF COLUMNS

### CHAPTER I

### PRELIMINARY

The design of reinforced concrete structures resolves itself into two portions, the first of which is concerned with the properties of the material produced by the combination of steel rods and the concrete in which they are embedded, and the second of which deals with the calculation of the loads and bending moments which the various members have to resist.

The various theories which have been developed in regard to the former, based on a very large amount of experimental work with the results of which they are in excellent agreement, are not difficult to understand, nor is there any great difficulty in making the necessary calculations for the resistance of beams to bending moments and other straining actions, and it is believed that Part I of the first volume of *Reinforced Concrete Design*, together with the additional formulae for the resistance of concrete beams to shear which have now been added in the present volume, is not only sufficiently clear to enable anyone possessing sufficient knowledge of the principles underlying engineering design in general to follow readily, but also contains so many curves and tables specially calculated to reduce the arithmetical work involved to an absolute minimum as to obviate the need of amplifying this portion of the subject.

The author's experience with technical students both at the University and in his own drawing office and with designs submitted to him by other designers for approval convinces him that most designers have little difficulty in determining the proportions of a member to resist given straining actions, but have, as a rule, very considerable difficulty in determining with any reliance the proper bending moments to be designed for.

Reinforced concrete design requires an intimate knowledge of the behaviour and properties of continuous beams, which is admittedly a complicated subject, and an attempt will therefore be made in the following chapters not only to explain in considerable detail what these complicated bending moment curves mean, how they are derived and how they are to be used, but also to work out and present a considerable number of curves and tables from which, with the explanations given, the design in any particular case may be resolved into reference to a simple curve or table, so as to eliminate a great deal of graphical and arithmetical work on the part of the designer.

Before embarking on this it will avoid confusion to explain in simple terms the meanings of words which are frequently used subsequently, in some cases in a special technical sense, in a way which may not always be obvious.

**Live Load and Dead Load.** In this book "live load" will be taken as denoting that portion of the total load on a floor or structure which may be applied unequally to the different portions of the structure, and "dead load" will imply that portion of the load which is either permanent, such as the weight of the structure itself, or which for some special reason will always be uniformly applied to the different portions of the structure in some definite proportion.

As an example of what is meant, the floor load for which an office building might be designed might be made up as follows:

| | |
|---|---|
| *Dead load*— | |
| Weight of reinforced concrete floor ... ... | 60 lbs. per sq. ft. |
| Weight of floor finish and plaster ... ... | 20 lbs. ,, |
| Weight of partitions ... ... ... ... | 20 lbs. ,, |
| *Live load*— | |
| Allowance for people, office fittings, etc. ... | 100 lbs. ,, |
| Total ... | 200 lbs. ,, |

Of this 200 lbs. the first three items may be taken to apply uniformly on the various panels and beams, whereas the last item of 100 lbs. per sq. ft. is an allowance which might very well, in some particular condition, exist over the whole of one panel while the adjoining panel was unloaded in this respect. In a warehouse, also, it is quite common for some of the panels to be loaded up to the ceiling with goods while the intermediate

panels between them may be quite unloaded so as to provide corridors for access, etc.; another time it may well be that a panel which at first was fully loaded now becomes unloaded (except for the dead load due to its own weight, etc., which of course persists), while an unloaded panel may become a loaded one.

It is clear that a beam of two spans, continuous over the central support, may have live load on one span and not on the other, or both spans may be fully loaded.

In ordinary steelwork design with free beams, with which the student is likely to be familiar, the question as to whether the second span is loaded or not has no effect on the bending moments in the first span, and therefore each span is treated independently, with the natural result that the only condition worth worrying over is that of each bay fully loaded.

Why cannot we do so in the design of reinforced concrete? Because in continuous beams, the condition of some spans unloaded often gives worse conditions of stress in parts of the beams than when all are fully loaded.

The student unfamiliar with this should master it, before worrying about how the moments are calculated, since till he has done so he cannot grasp the reason for this admittedly more confusing and difficult treatment than suffices for steelwork. Let the student refer to Fig. 4. The top curve (*a*) shows the bending moments in a continuous beam of two spans, both bays loaded. The bending moment if there were no continuity would of course be $+\frac{Wl}{8}$ at mid-span and zero at the supports, but the effect of continuity is to reduce the maximum moment near mid-span to $\frac{Wl}{14{\cdot}4}$ (just over half its previous value) and to produce a negative bending moment of $-\frac{Wl}{8}$ at the middle support.

What happens is that part of the beam near the centre support acts as a balanced cantilever—rather like a part of the Forth Bridge—and the remaining parts of the beam therefore act as simple beams of a lesser span, carried at the inside end on one end of the cantilever referred to. The cantilever

portion is bent in such a way as to produce tension on top and compression below—the condition we associate with negative moments—while the simple beams referred to have tension below and compression above (positive moments).

Now it is clear that with both spans loaded the cantilever action is most effective, and this condition is that under which the moment at the centre support is greatest, and its value under this condition is what determines the moment that we design the support portion of the beam for.

When now one span is loaded and the other span unloaded, the cantilever would be unbalanced if its length were constant, and would rock on the centre support and let the loaded span drop. This is prevented by designing the unloaded span (and either may in turn be the unloaded one) strong enough to act as a cantilever right out to the end support, and so resist the tendency to rock. At the same time, conditions of elasticity produce the effect that the tendency to rock is further reduced by the length of the cantilever on the side of the loaded panel becoming less, and the length of the effective span of the simple beam in the loaded span becoming greater. In this condition, the positive moment in the loaded span is therefore greater than it was with both spans loaded, and while the negative moment at the support is now less than it was (one-half) it now extends all along the unloaded span, which before was mostly stressed with positive moments. Clearly, then, both these conditions of loading must be designed for.

With more than two spans, the different conditions which produce the greatest straining actions in different parts are greater in number. With many equal spans, it may be shown that the worst case for positive moments near mid-span in the loaded bays and for negative moments near mid-span in the unloaded spans is when alternate spans are loaded. The worst case at the supports is when the two spans either side of any particular support are loaded, and the next panels unloaded, and so on.

Now in constructional steelwork, it would not matter much whether the moment were positive or negative, since both flanges are equally well adapted to resist tension or compression. But in concrete design it is quite different. The concrete is

generally able to resist the compression forces, but steel has to be provided where tension may occur. Therefore negative moments need provision of steel in the top, and positive moments steel in the bottom of a beam, and an accurate knowledge of the worst positive and negative moments at all points along a beam is therefore an essential precedent to the design of an economical beam where steel is only provided where, and to the extent, that is necessary.

It will be clear from the foregoing that with live load the bending moments to be provided for are greater, both positive and negative, than with dead load, since with the former we have to consider the worst of several conditions of loading, and with the latter one condition only, and that, one which produces neither near mid-span nor near supports, so great a moment as for live load.

Clearly then, live load means more steel to provide than dead load.

Now, in a practical case, part of the load is always dead, and it is economical to take advantage of this. For example, near mid-span, the live load produces negative moments in unloaded panels, while the dead load produces positive moments which often go a long way towards neutralising them for the centre portion of the span, and so obviate the need for providing top steel here.

This, unfortunately, means yet another complication in design, but in the interests of economy it is one which has to be faced. In the present work it will be found that curves are given for the live load and dead load combined for a large range of ratios of live to dead, for several common load distributions, and any number of spans, and the series of curves has been made complete enough to enable *any* practical case of equal spans to be solved by simple reference to the appropriate curve.

**Parabolas.** The student will be well advised to calculate and draw some of these curves himself, so as to grasp the subject and come into contact with some of the difficulties. In this, and also when dealing with unequal spans, he will often need to construct parabolic bending moment curves.

Fig. 1 will be found convenient as giving the ordinate at

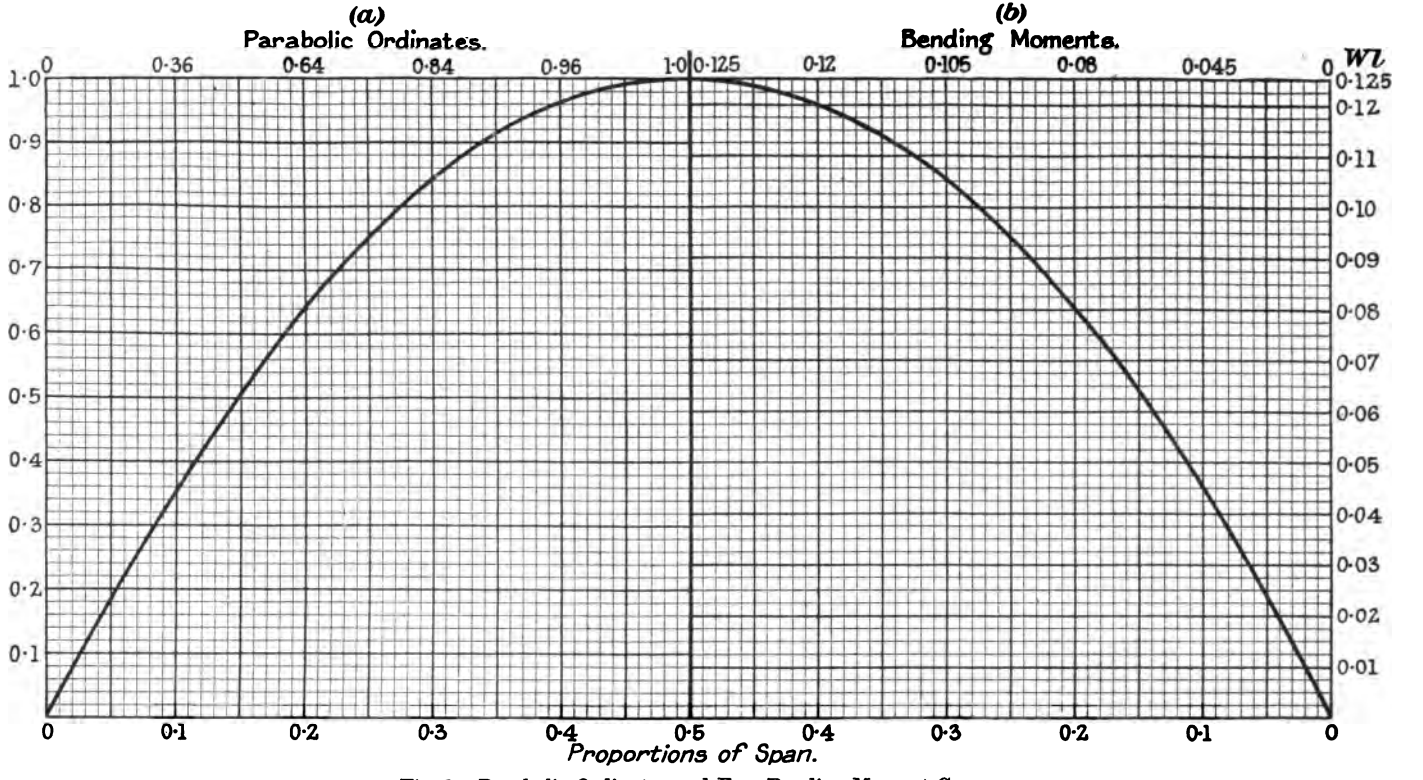



Fig. 1. Parabolic Ordinates and Free Bending Moment Curve.

any proportion of the span, expressed both as a fraction of the whole diagram as it would be if there were no continuity—henceforth called *the free moment diagram*—or in terms of $Wl$.

Sometimes it is convenient to construct parabolas graphically. The student is probably familiar with the method shown in Fig. 2, which is most convenient, and applies equally to a parabola erected on a horizontal or a sloping base, as is generally required for bending moment curves.

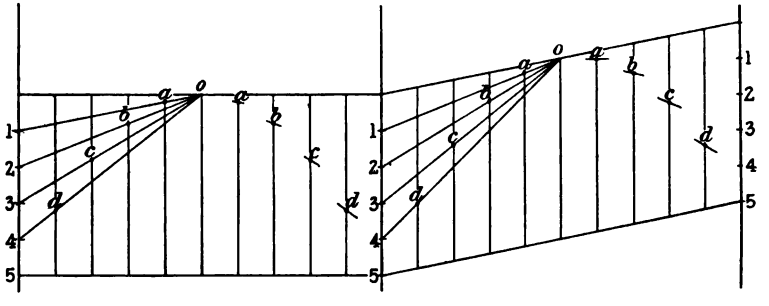



Fig. 2. Graphical Method of constructing Square and Skew Parabolas.

The construction consists in setting up at mid-span a vertical ordinate representing the free moment, dividing the half span, and the height of the diagram into the same number of equal parts—five generally does, sometimes less—raising vertical lines on the divisions of the base, and joining the top of the ordinate $o$ raised at mid-span to the divisions 1, 2, 3, 4 and 5, of the ordinate at the support.

Where $o4$ cuts the vertical line at ·1 along the span represents a point $d$ on the parabola, similarly the intersection of

$o3$ at ·2 gives $c$,
$o2$ at ·3 gives $b$,
$o1$ at ·4 gives $a$.

These points may then be joined by an even curve.

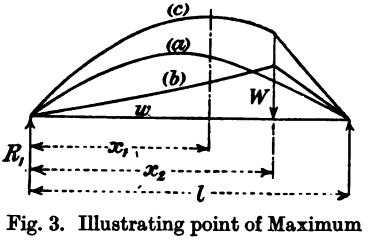



Fig. 3. Illustrating point of Maximum Moment of Combined Curves.

It is useful, when using the theorem of three moments, to remember that the area of a parabola is two-thirds that of the enclosing rectangle.

**Maximum Moment with Combined Loadings.** The following little theorem is often convenient for determining at what point along a beam will be the maximum moment when there is both uniform and point load to design for. Thus in Fig. 3 we have a moment curve (*a*) due to uniform load and (*b*) due to point load. The combined curve (*c*), obtained by adding (*a*) to (*b*) at every section, will have a maximum value at a distance $x_1$ from the reaction $R_1$ determined by

$$x_1 = \frac{R_1}{w},$$

where $R_1$ is the total reaction due to both loadings, $w$ is the load per unit length of the distributed load. If $x_1$ works out greater than $x_2$, the position of the point load, then $x_2$ is the position of maximum moment.

We use for this formula the reaction farther from the point load, and measure our distances from it. The truth of the formula is obvious when it is remembered that the place of maximum moment is the place of no shear.

Hence $$R_1 - x_1 w = 0,$$

and $$x_1 = \frac{R_1}{w}.$$

This formula is only strictly accurate for beams in which either there is no continuity, or the reverse moments at the ends are equal.

## CHAPTER II

### A CONTINUOUS BEAM OF TWO EQUAL SPANS WITH UNIFORMLY DISTRIBUTED LOADING

Figure 4 (*a*) shows the bending moment curve for both spans loaded which, of course, is the condition that always applies to dead and sometimes applies to live load. The bending moment at the central support is $-\frac{Wl}{8}$ and at mid-span is $+\frac{Wl}{16}$ but increases to $\frac{Wl}{14{\cdot}4}$ at a distance of about $\cdot 4l$ from the outer supports.

The value of the moment at the centre support may be derived from Appendix I of Volume I of *Reinforced Concrete Design*, or may be immediately derived from the theorem of three moments which is given and explained later.

Notice the very considerable economy which results in the case of the dead load by designing the beam as a continuous one. The bending moment for the greater length of the beam is reduced from $\frac{Wl}{8}$ to $\frac{Wl}{14{\cdot}4}$ and only at the centre support is a large moment produced which can be resisted by a haunch between the beam and the column at a very small cost.

Coming now to Fig. 4 (*b*), this gives a moment curve with the left-hand bay loaded and the right-hand bay unloaded, a condition which may obviously occur in practice. It will be seen that on the loaded panel the positive bending moment is considerably greater than it was for the condition of both bays loaded and this therefore is the positive bending moment which needs to be designed for as regards the live load portion of the total load to which the beam may be subjected.

The right-hand portion of the diagram, the unloaded portion, is equally important because the diagram indicates that the unloaded span may be subjected to a negative moment throughout its length varying from $-\frac{Wl}{16}$ at the centre to nothing at

the outer support, whereas with both spans loaded no such condition had to be provided against.

Now clearly though diagram 4 (*b*) is drawn with the left-hand bay loaded and the right-hand unloaded this condition

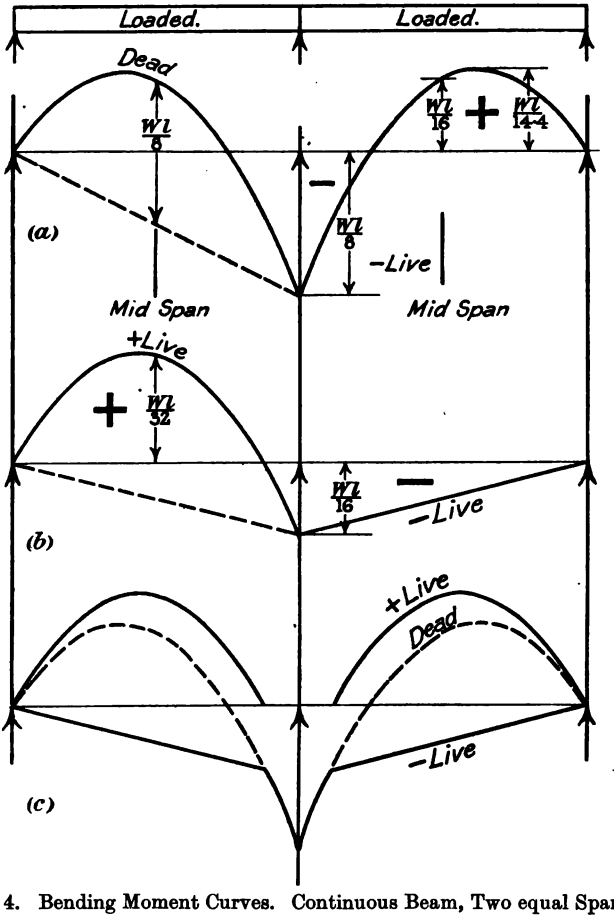



Fig. 4. Bending Moment Curves. Continuous Beam, Two equal Spans. Different Conditions of Loading.

may easily be reversed and therefore both sides of the diagram have to be designed against both curves, positive and negative.

Fig. 4 (*c*) shows the curve of maximum bending moments combined from the two upper ones. The dead load curve is shown dotted, the positive live load curve is derived from the

UNIV. OF



CALIF.

left-hand portion of Fig. 4 (*b*) and the negative live load curve from the right-hand portion of 4 (*b*). The cusp at the centre support is derived from the top diagram of Fig. 4 (*a*), and for two spans the negative cusp for live load happens to be the same as for dead load, whereas with more spans they are different, as we shall see later.

Now clearly if part of the load is dead it is more economical to design for it in accordance with the dead load curve and only apply the larger bending moments which the live load requires to the actual live load. It is because of this important economy that we have to go to the trouble and complication of designing for the live load and dead load separately.

Fig. 5 is one-half of Fig. 4 (*c*) enlarged, from which values at any point along the span may be seen at a glance and tabulated values are also given. With this diagram it is clearly quite easy to see what the bending moment at any section of the beam may be. As an example, if we consider a point at $\cdot 4l$ from the outer support, the worst live load to design for is clearly $\cdot 095Wl$ where $W$ is the live load, to which must be added $\cdot 07Wl$ where $W$ is the dead load. As regards the negative moments at this point, that due to the live load may amount to $-\cdot 025Wl$, from which, however, has to be subtracted $+\cdot 07Wl$ due to the dead load. It will be seen that where the dead load is more than one-third of the live load the positive moment more than counterbalances the negative one, in which case no negative moment needs to be designed for at this particular point.

To make these curves still more valuable and more readily available without any calculation Figs. 6 and 7 are given in which the live load and dead load have been combined in the way which was explained for the one point. Thus, Fig. 6 (*a*) is the curve to design for when all the load is dead.

It will be remembered that we use the word "dead" in the special sense of a load which cannot be imposed on one span without being equally imposed on the other at the same time. Therefore, for example, a water tower of two spans with no central partition may be treated as subjected to dead load because whether the tank is full or empty or partly full both spans will be equally loaded, and a beam in such a tower may safely be designed in accordance with Fig. 6 (*a*).

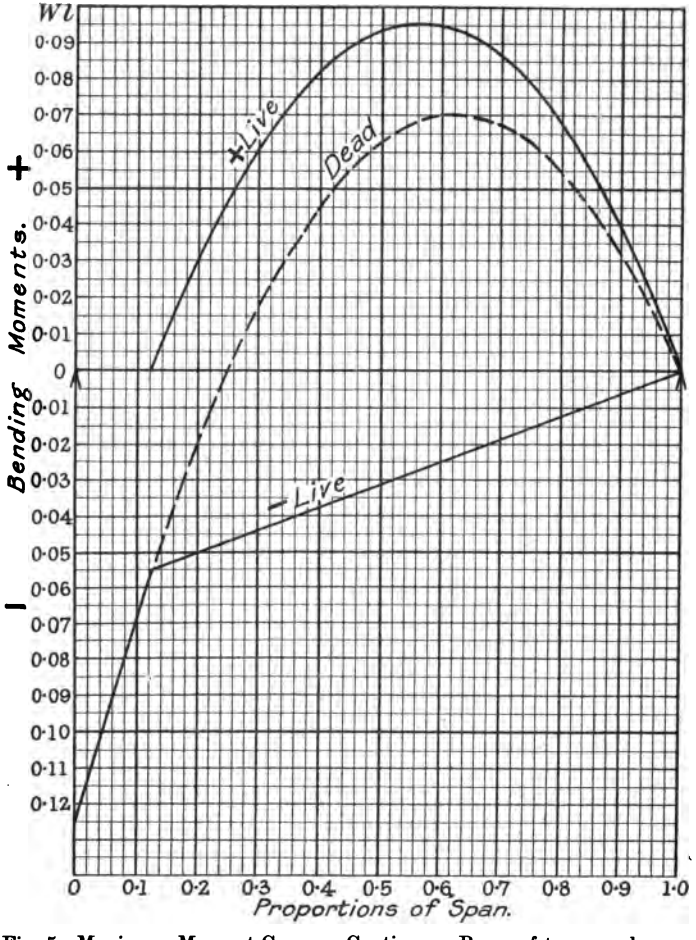



Fig. 5. Maximum Moment Curves. Continuous Beam of two equal spans.

Tabulated Values of Moments.

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| +live | [−·0625] | [−·0112] | −·03 | ·0612 | ·0825 | ·0937 | ·095 | ·0862 | ·0672 | ·0387 | 0 |
| dead | −·125 | −·0675 | −·02 | ·0175 | ·045 | ·0625 | ·07 | ·0675 | ·055 | ·0325 | 0 |
| −live | −·125 | −·0675 | −·05 | −·0437 | −·0375 | −·0312 | −·025 | −·0187 | −·0125 | −·0062 | 0 |

UNIV. OF CALIFORNIA

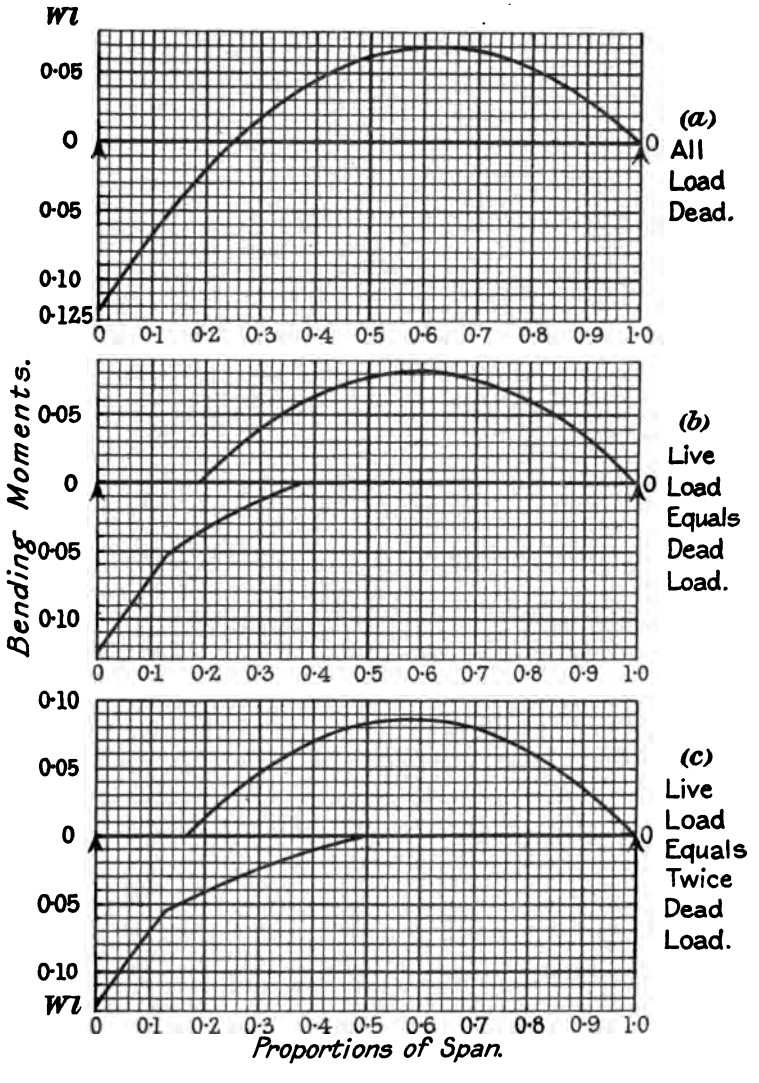



Fig. 6. Maximum Moment Curves. Continuous Beam of two equal spans. Definite Ratios of Live Load to Dead Load.

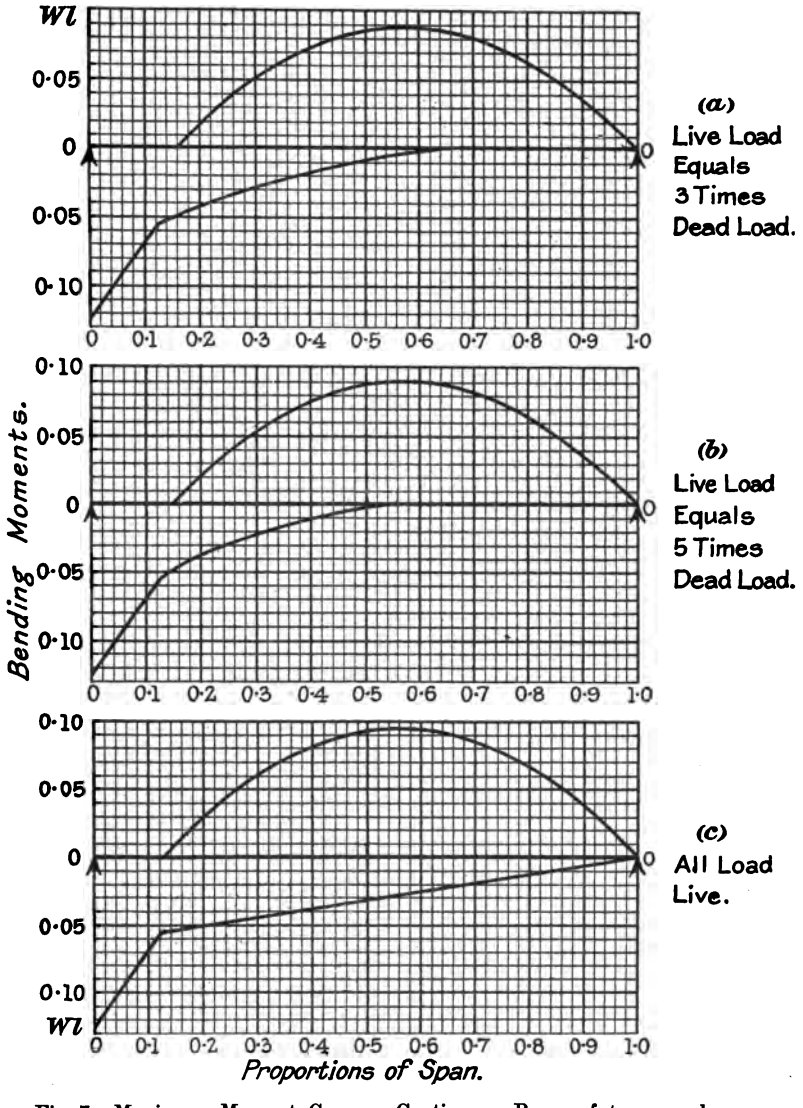



Fig. 7. Maximum Moment Curves. Continuous Beam of two equal spans. Definite Ratios of Live Load to Dead Load.

Fig. 6 (*b*) shows the maximum moments that need to be designed for when live load and dead load are equal, a condition which nearly always applies approximately for floors designed for office loads. Here it will be seen that the positive moment extends to about $\cdot 19l$ from the centre support and its maximum value is ·083. The negative moment extends from the centre support to a distance of $\cdot 37l$ from it and beyond this there is no negative moment to provide for. Similarly Figs. 6 (*c*) and 7 (*a*), (*b*) and (*c*) indicate at a glance what moment each section needs to be designed for from which the top and bottom steel can be calculated immediately. The value of this is very great as it enables the work to be done with great accuracy without entailing a lot of extra labour on the designer.

Remember that when designing for two spans the effect of continuity over the centre support is to increase the reaction on the centre support and to reduce it on the two outer supports. This was explained in *Reinforced Concrete Design*, Vol. I. With both spans loaded the centre reaction amounts to $\frac{5}{4}W$, which it will be seen is 25 per cent. greater than if non-continuous beams had been adopted, as in steel work, for example. This increase is clearly too great to be neglected and must certainly be taken into account when designing the centre columns between continuous beams of two spans.

With both bays loaded the reactions at the outer supports are $\frac{3}{8}W$, which is 25 per cent. less than if non-continuous beams were used, but with one span only loaded this reaction increases to $\frac{7}{16}W$, which is only $12\frac{1}{2}$ per cent. less than the reaction when non-continuous beams are used and this is therefore the value to design for as regards the live load while the outer supports may be designed for $\frac{3}{8}W$ as regards the dead load.

If the end supports are reinforced concrete columns, these will cause some restraint at the ends and produce negative moments at the ends. The value of this can be calculated as explained in Part II. Enough steel in such cases must be provided in the top at the ends to take such moments safely, otherwise cracks will occur here which may develop into dangerous shear failure.

# CHAPTER III

## A CONTINUOUS BEAM OF TWO SPANS, FREE AT THE OUTER SUPPORTS, LOADED WITH A SINGLE POINT LOAD AT MID-SPAN

Fig. 8 shows an arrangement of beams which is often adopted in factory construction when the column spacings are about 15 ft.–20 ft. square, and such panels are designed by considering the slab as transferring the load to the secondary beams, which, in their turn transmit the load of the floor to the main beam and concentrate it at the centre of the main beam.

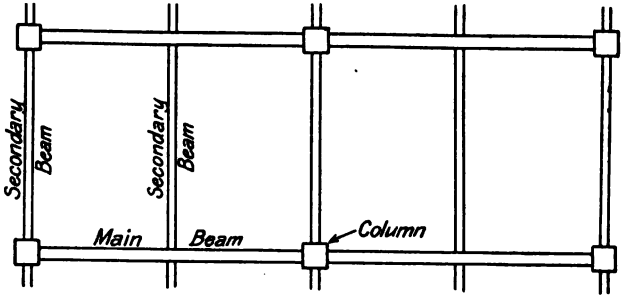



**Fig. 8. Arrangement of Beams in floor producing effect of Central Point Loads on Main Beams.**

This is the most frequent application of the subject-matter of this chapter though its use is also considerable in regard to rolling loads, as for viaducts, etc., which will be considered in a later chapter.

If $W$ represents the total load acting at mid-span, the bending moment if the ends are free would be $\frac{Wl}{4}$, and as was explained more fully in Chapter II, the bending moments for continuous beams become fully determined if we can calculate the negative bending moment at central support, knowing as we do that the bending moment at the two extreme ends is zero as the beams are assumed free in these points. Freedom at these points is generally accurate enough for practical pur-

poses and on the side of safety, since the restraint afforded by a brick wall is generally negligible, and where the outside support is a reinforced concrete column monolithic with the beam, the bending moment at the end is quite small in comparison with those in the beam when the columns are relatively small as occurs in the upper storys of a building, but gradually increases as the columns get bigger and stiffer lower down.

*Determination of Span.* For the purposes of beams resting on walls some slight allowance for the restraint may be made by considering the span as being from the centre of the middle support or column to the inside face of the outer wall, not the centre of the bearing on the outer wall, and this same convention may be used where the outside columns are relatively slender. Where the outside columns are, however, relatively stiff, as they may be near the bottom of a high building, and it is desired to take this restraining bending moment on the outside columns into account in designing the beams, this moment may be calculated in accordance with the treatment of formulae given in Vol. I, *Reinforced Concrete Design*, pp. 143–155, but more conveniently read off from Figs. 64 and 65 of the present volume. Though the treatment here was intended to apply more to the column than to the beam, the calculation of the bending moment at this point obviously applies just as well to the design of the beam. When this has been calculated it will be found to be relatively small except for very stiff columns, and the bending moment diagrams, Figs. 9 to 12, may still be used by setting out on the line of the outer support the bending moment equal to the one calculated as described, joining this point to the point where the base line of the bending moment diagram intersects the line of the centre support, and using this as a new base line for reading off the bending moments. This will not be absolutely accurate as the negative moments at the central support will be rather too high, but with the small bending moments which are developed in the outside columns in ordinary cases, the error involved is very small and the convenience of the method very considerable. Do not forget in this connection that the outside column must be designed to carry this bending moment in addition to its direct load.

In the following calculations the ends of the beam are assumed free.

**Calculation of Bending Moment on Central Support.** The calculation of this bending moment is by no means a simple matter and involves relatively difficult questions of elasticity. The theorem of three moments or the theorem of last work may be used.

The ensuing formulae are, however, worked out to a certain point in Vol. I of *Reinforced Concrete Design*, Appendix I, 3, p. 269.

It is, however, not obvious to many people how to use these formulae as they stand and the treatment is therefore given in full, as follows.

It will be remembered that:

$E$ is the modulus of elasticity of the material of which the beam is constructed,

$B$ is the moment of inertia of the beam divided by its span,

$a$ is the slope of the beam at the support produced by deflection under the load, expressed in circular measure, or as the tangent to the angle as these slopes are infinitesimal.

As, however, all these quantities cancel out and give us a simple formula in which they do not appear, it is unnecessary to worry at this stage as to exactly how they are calculated as long as it is understood what they denote.

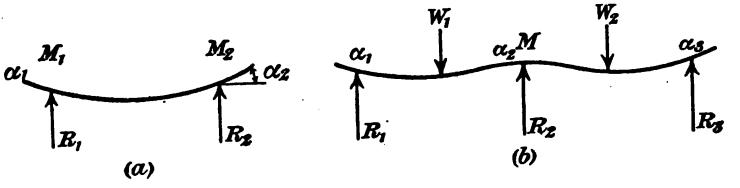



Fig. 9. Continuous Beam of two spans, Point Loads at mid-span, notation.

From *Reinforced Concrete Design*, Vol. I, Appendix I, 3, p. 269, equations (7) and (8) give (Fig. 9 (*a*)):

$$M_1 = -\frac{Wl}{8} - 4EBa_1 - 2EBa_2$$

$$M_2 = -\frac{Wl}{8} + 4EBa_2 + 2EBa_1$$

Applying to left-hand span of Fig. 9 (*b*):

$$O = -\frac{W_1 l}{8} - 4EBa_1 - 2EBa_2 \quad . \ . \ . \quad (1)$$

$$M = -\frac{W_1 l}{8} + 4EBa_2 + 2EBa_1 \quad . \ . \ . \quad (2)$$

and to right-hand span

$$M = -\frac{W_2 l}{8} - 4EBa_2 - 2EBa_3 \quad . \ . \ . \quad (3)$$

$$O = -\frac{W_2 l}{8} + 4EBa_3 + 2EBa_2 \quad . \ . \ . \quad (4)$$

From (1) $$4EBa_1 = -\frac{W_1 l}{8} - 2EBa_2$$

Substituting this in (2)

$$M = -\frac{W_1 l}{8} + 4EBa_2 - \frac{W_1 l}{16} - EBa_2$$

$$= -\tfrac{3}{16} W_1 l + 3EBa_2 \quad . \ . \ . \ . \ . \ . \ . \quad (5)$$

From (4) $$-4EBa_3 = -\frac{W_2 l}{8} + 2EBa_2$$

Substituting this in (3)

$$M = -\frac{W_2 l}{8} - 4EBa_2 - \frac{W_2 l}{16} + EBa_2$$

$$= -\tfrac{3}{16} W_2 l - 3EBa_2 \quad . \ . \ . \ . \ . \ . \ . \quad (6)$$

From (5) and (6)

$$-\tfrac{3}{16} W_1 l + 3EBa_2 = -\tfrac{3}{16} W_2 l - 3EBa_2$$

whence $$6EBa_2 = \tfrac{3}{16} l (W_1 - W_2)$$

Whence, from (5)

$$M = -\tfrac{3}{16} W_1 l + \tfrac{3}{16} l \left(\frac{W_1}{2} - \frac{W_2}{2}\right)$$

$$= -\tfrac{3}{32} l (W_1 + W_2) \quad . \ . \ . \ . \ . \ . \ . \quad (7)$$

When both bays loaded with $W$, this gives

$$M = -\tfrac{3}{16} Wl \quad . \ . \ . \ . \ . \quad (7a)$$

This is the basis for construction of the bending moment diagrams for dead load, and for live load, both spans loaded.

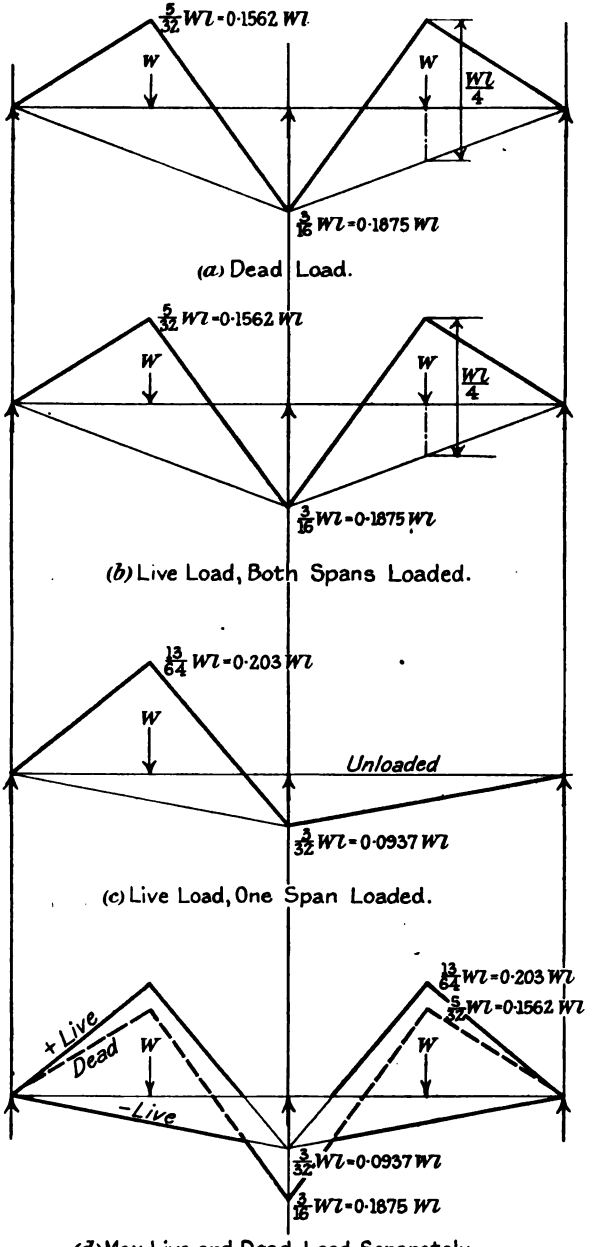



**Fig. 10. Curves of Moments. Two Spans. Point Loads at mid-span. Different conditions of loading.**

When one bay only is loaded with $W$, this gives

$$M = -\tfrac{3}{32}Wl \quad . \quad . \quad . \quad . \quad . \quad . \quad (7b)$$

This is the basis for the construction of bending moment diagrams for live load, one span only loaded.

Note, in passing, that the maximum reaction on centre support

$$\begin{aligned} R_2 &= (W_1 + W_2) - R_1 - R_3 \\ &= W_1 + W_2 - \frac{W_1}{2} - \frac{M}{l} - \frac{W_2}{2} - \frac{M}{l} \\ &= \frac{W_1 + W_2}{2} - \frac{2M}{l} \\ &= \frac{W_1 + W_2}{2} + \tfrac{3}{16}(W_1 + W_2) \\ &= \tfrac{11}{16}(W_1 + W_2) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8) \end{aligned}$$

The outer reaction

$$\begin{aligned} R_1 &= \frac{W_1}{2} - \tfrac{3}{32}(W_1 + W_2) \\ &= \tfrac{13}{32}W_1 - \tfrac{3}{32}W_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (9) \end{aligned}$$

In other words, with both spans loaded, outer reaction is

$$R_1 = \tfrac{10}{32}W = \tfrac{5}{16}W \quad . \quad . \quad . \quad . \quad . \quad (10)$$

and with only one span loaded, outer reaction of loaded span is

$$R_1 = \tfrac{13}{32}W \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

and outer reaction of unloaded span is

$$R_1 = -\tfrac{3}{32}W \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

that is, producing tension in the column under it to that amount.

Selecting the worst cases of these results, and expressing as live load and dead load, we have

Max. moment at centre support

$$M = -\tfrac{3}{16}W_l l = -\tfrac{3}{16}(W_l + W_d)\,l \quad . \quad . \quad . \quad (13)$$

Max. reaction at centre support

$$R_2 = \tfrac{11}{16}(W_l + W_l) = \tfrac{11}{8}W_l = \tfrac{11}{8}(W_l + W_d) \quad . \quad (14)$$

Max. reaction at outer support

$$R_1 = \tfrac{13}{32}W_l - \tfrac{3}{32}W_d = \tfrac{13}{32}W_l + \tfrac{5}{16}W_d \quad . \quad . \quad (15)$$

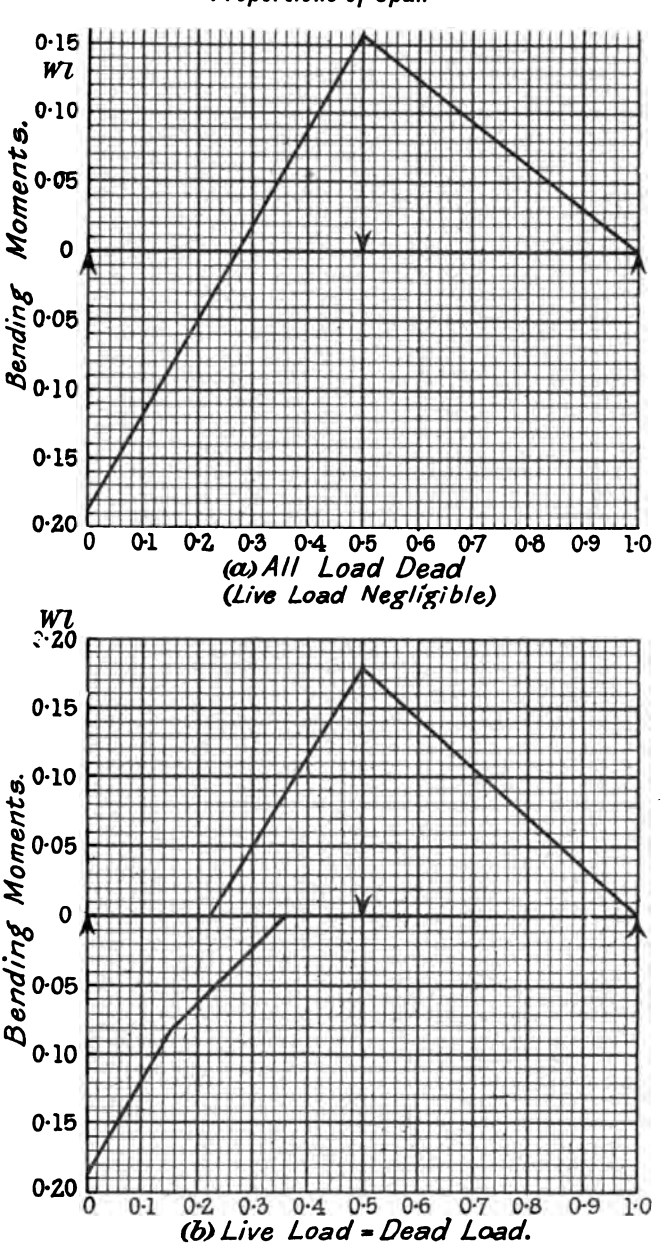



Fig. 11. Maximum Moment Curves. Two Spans. Central Point Loads. Definite Ratios of Live Load to Dead Load.

Min. positive reaction or max. negative reaction at outer support

$$R_1 = \tfrac{13}{32}W_d - \tfrac{3}{32}W_l = \tfrac{5}{16}W_d - \tfrac{3}{32}W_l \quad . \quad . \quad . \quad (16)$$

If this gives a negative result, the reaction is upward.

Having now calculated the bending moment on the central support under different conditions of loading, it becomes an easy matter to draw the various bending moment diagrams. In Fig. 10 (*a*) we set down the bending moment $\frac{3}{16}Wl$ (see Formula 7 (*a*)) and join this point to the point where the base line intersects the two outer supports. On this sloping line we erect the usual bending moment diagram for a beam with a single central support, producing a central moment of $\frac{Wl}{4}$. The resultant diagram is a bending moment with a central load of $W$ on each span, and clearly applies to the condition of dead load, and also to the condition of live load when both spans are loaded (Fig. 10 (*b*)). It will be seen that the maximum moment at mid-span is $\frac{5}{32}Wl$. In Fig. 10 (*c*) we set down a moment on the central support of $\frac{3}{32}Wl$ (see Formula 7 (*b*)), join this point to the point where the base line of the diagram intersects the outer support, and on this line erect the usual bending moment diagram as if the ends were free. This gives us the moment on the left-hand, or loaded, span. On the right-hand span there is no load and therefore no diagram to build up on our sloping base line, which therefore remains as a negative bending moment diagram for the unloaded bay. This negative bending moment along the span is one to which insufficient attention is frequently given, but it will be seen that its values are by no means negligible.

In Fig. 10 (*d*) the conditions of the three previous diagrams are considered and the worst cases selected at each section of the beam. Thus for the positive live load diagram we do not use Fig. 10 (*b*) [both spans loaded], but Fig. 10 (*c*) [only one span loaded], because it gives us bigger bending moments. For the negative live load bending moment diagram we select the right-hand diagram of Fig. 10 (*c*), except near the central support, where we find that worse conditions are given on the lower cusp of Fig. 10 (*b*) [both spans loaded], which is therefore used until it intersects the other negative live load line.

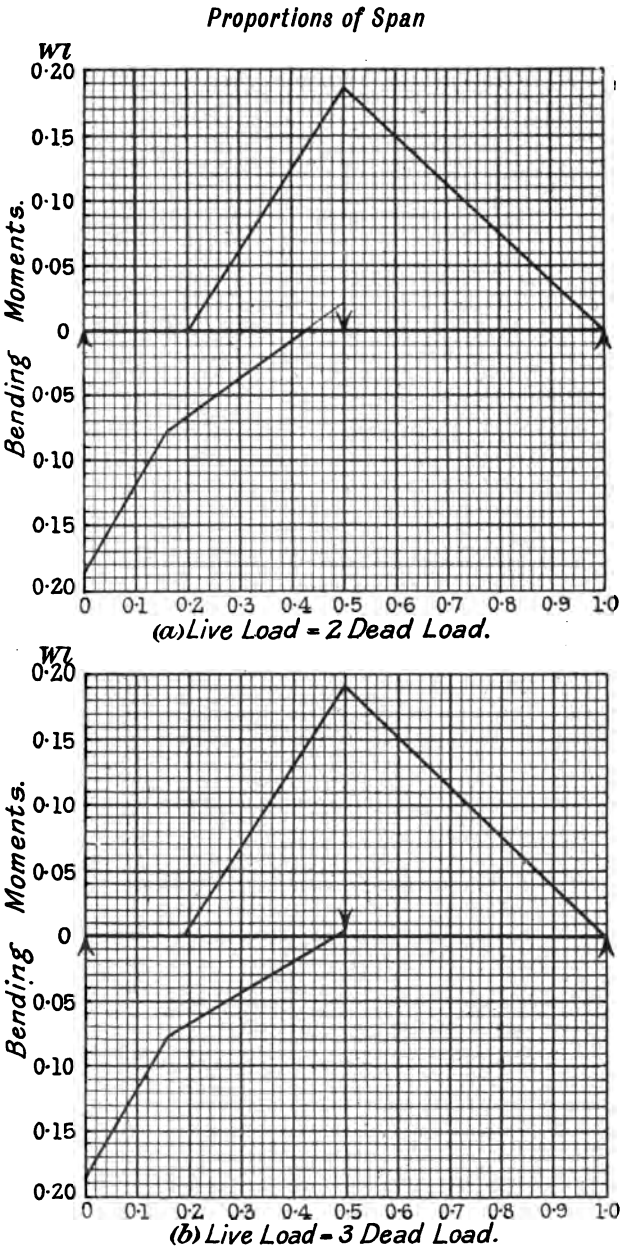


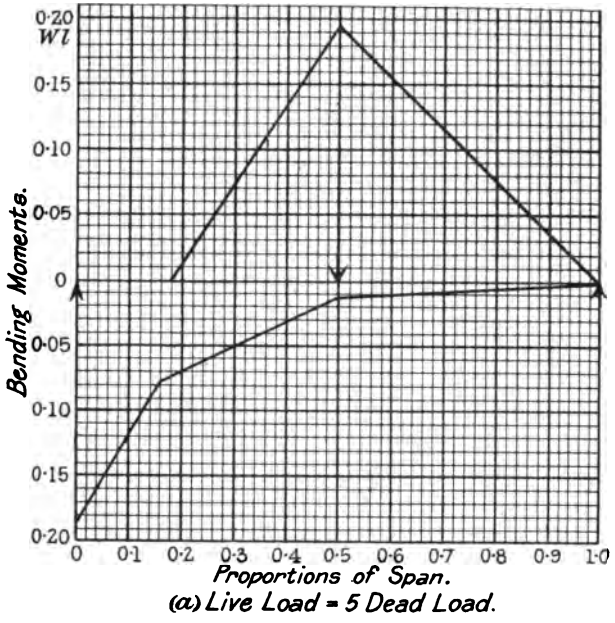

Fig. 12. Maximum Moment Curves. Two Spans. Central Point Loads. Definite Ratios of Live Load to Dead Load.

In addition to these two, the dead load curve is inserted from Fig. 10 (*a*) and shown dotted. The necessity for showing this, although it gives lesser results than the positive live load curve, arises from the fact that in practice part of the load only is live and the rest dead and economy is therefore obtained by applying the lower values of the dead load curve to the dead load and only using the higher values of the live load curve to the live load.

In Fig. 10 (*c*) with one span loaded the positive moment occurs on the left span and the negative moment on the right span; nevertheless, it is obvious that in practical examples the right-hand bay might just as easily be loaded and the left-hand bay unloaded so that Fig. 10 (*c*) would be reversed, end for end, and both cases are, of course, considered in compiling Fig. 10 (*d*), which gives the maximum bending moments, positive or negative, to which any section of the beam might be exposed.

Figs. 11 (*a*) and 13 (*b*) are enlarged diagrams of the right-hand span of Fig. 10 (*d*). Fig. 11 (*a*) being the dead load curve and Fig. 13 (*b*) being the positive and negative live load curves.

In practice there is generally some live and some dead load to be considered and we may, if desired, calculate the dead load bending moments from Fig. 11 (*a*) and the live load bending moments from Fig. 13 (*b*) and add these together for any particular section under consideration, having due regard to signs. This is, however, a relatively tedious process and generally means that in practical work the student is sorely tempted to calculate only the bending moments at the support and mid-span and to leave the intermediate ones to chance. This is extremely undesirable because the writer has frequently found students doing this and not providing for negative moments which produce tension at the top, not knowing properly where the various bars may be stopped off with safety, and altogether this leads to much uncertainty with risk of danger or extravagance. Diagrams in which live load and dead load have been combined are therefore given in Figs. 11 (*b*) to 13 (*a*) and may be used in all practical cases without further calculation and give a very graphical picture of what may happen at all points along the span and exactly what moments must be provided



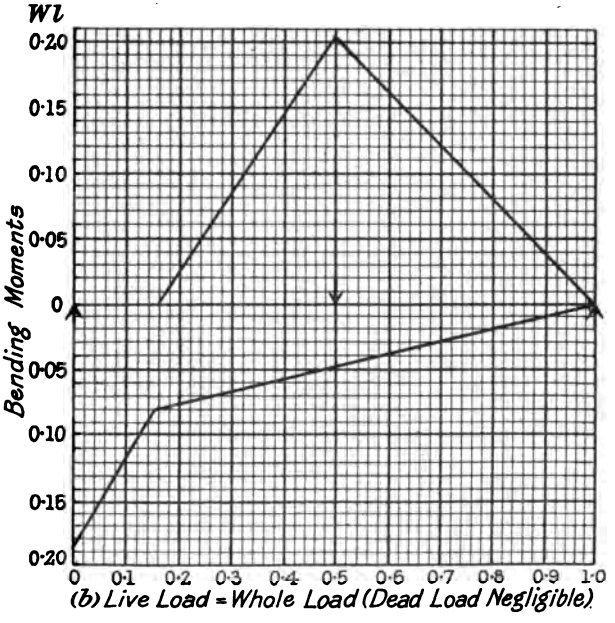




Fig. 13. Maximum Moment Curves. Two Spans. Central Point Loads. Definite Ratios of Live Load to Dead Load.

for at all points. With these diagrams the steel required at the top and bottom of the beam at any point along the span can be calculated with the very greatest ease.

It will be seen that any two adjacent diagrams differ very little from each other and any intermediate case can therefore readily be obtained with great accuracy, and very little lack of economy will result from taking in all cases the lower diagram, i.e., for the case of live load equals four times dead load, for example, using Fig. 13 (*a*), and this course is always on the side of safety.

The reader is referred to the end of Chapter II for the treatment of negative moments at the end supports where these are monolithic.

# CHAPTER IV

## A CONTINUOUS BEAM OF TWO SPANS, CONTINUOUS OVER A CENTRAL SUPPORT, FREE AT THE OUTER ENDS, LOADED WITH TWO POINT LOADS AT THE THIRD POINTS

This condition arises in practice chiefly in the case of floor panels arranged as in Fig. 14, which is perhaps the commonest arrangement in floors where the column centres are between 20 and 25 ft.

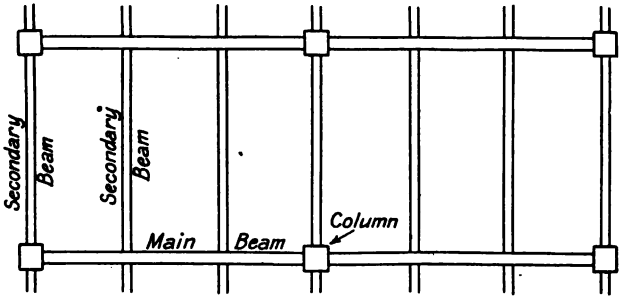



**Fig. 14. Arrangement of Beams producing Two Point Loads at Third Points of Main Beam.**

The notes in Chapter III as to how far the outer ends are really free in practice may, with advantage, be referred to, as also may the notes on how the diagrams may be corrected for those cases where it is desired to make allowance for small negative moments at the end supports. The arrangement of loading also frequently occurs in connection with moving loads on viaducts, etc., where, for example, the beams of the viaduct are 20 ft. span and trucks about 13 ft. 4 in. long, with a wheel base of about 6 ft. 8 in., figures which are frequently near the truth, it will be clear that the beams are often loaded in accordance with the heading to this chapter.

As was explained in Chapter III, no difficulty exists in drawing bending moment diagrams when we know the value of the negative bending moment on the central support, and this

will therefore be calculated in some detail, starting off with the basis formulae derived in *Reinforced Concrete Design*, Vol. I, Appendix I, 4, p. 272.

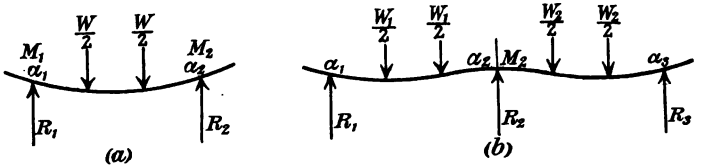



Fig. 15. Beam of Two Spans. Two Point Loads at Third Points. Notation.

From *Reinforced Concrete Design*, Vol. I, Appendix I, 4, p. 272,

$$M_1 = -\frac{Wl}{9} - 4EBa_1 - 2EBa_2 \text{ [notation of Fig. 15 } (a)\text{]}$$

and similarly $M_2 = -\frac{Wl}{9} + 4EBa_2 + 2EBa_1$

Applying to left-hand span of Fig. 15 (*b*),

$$O = -\frac{W_1 l}{9} - 4EBa_1 - 2EBa_2 \quad . \quad . \quad . \quad . \quad (1)$$

$$M = -\frac{W_1 l}{9} + 4EBa_2 + 2EBa_1 \quad . \quad . \quad . \quad . \quad (2)$$

and to right-hand span

$$M = -\frac{W_2 l}{9} - 4EBa_2 - 2EBa_3 \quad . \quad . \quad . \quad (3)$$

$$O = -\frac{W_2 l}{9} + 4EBa_3 + 2EBa_2 \quad . \quad . \quad . \quad (4)$$

From (1) $$4EBa_1 = -\frac{W_1 l}{9} - 2EBa_2$$

Substituting this in (2)

$$M = -\frac{W_1 l}{9} + 4EBa_2 - \frac{W_1 l}{18} - EBa_2$$

$$= -\tfrac{1}{6}W_1 l + 3EBa_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

From (4) $$-4EBa_3 = -\frac{W_2 l}{9} + 2EBa_2$$

Substituting this in (3)

$$M = -\frac{W_2 l}{9} - 4EBa_2 - \frac{W_2 l}{18} + EBa_2$$

$$= -\tfrac{1}{6}W_2 l - 3EBa_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

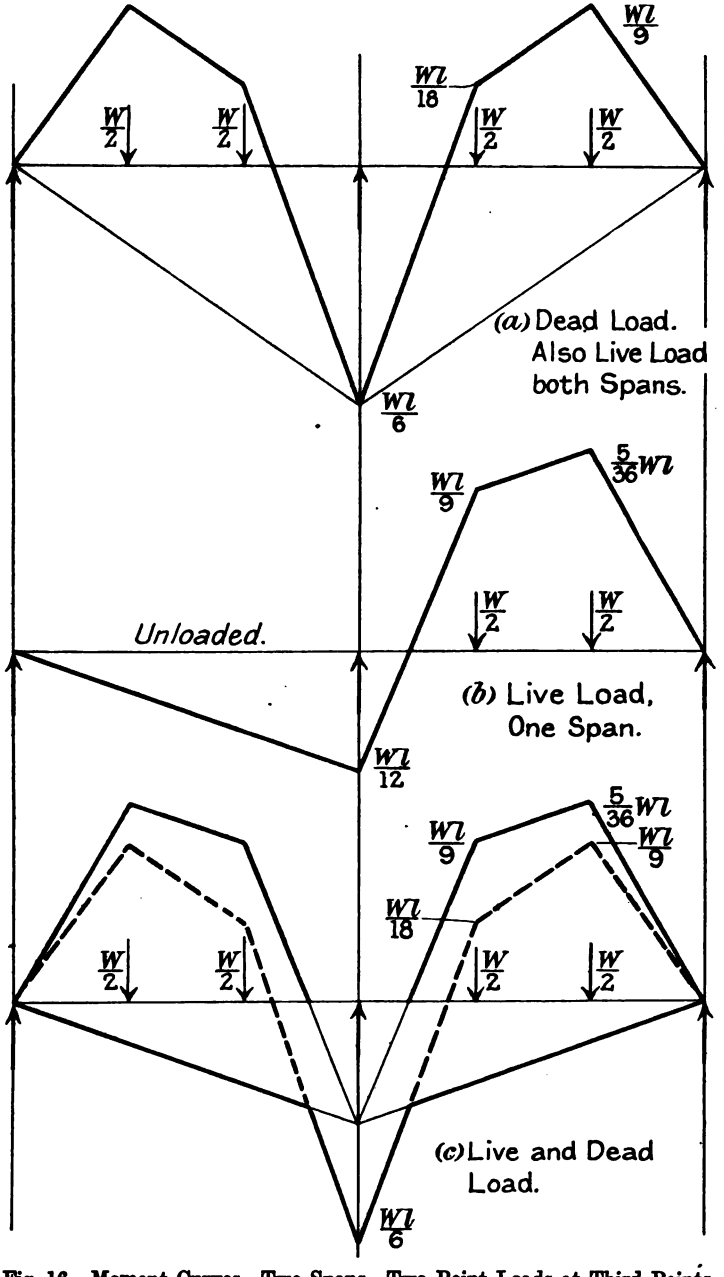



**Fig. 16. Moment Curves. Two Spans. Two Point Loads at Third Points. Live and Dead Load.**

From (5) and (6)

$$-\tfrac{1}{6}W_1 l + 3EBa_2 = -\tfrac{1}{6}W_2 l - 3EBa_2$$

whence $$6EBa_2 = \tfrac{1}{6}l(W_1 - W_2)$$

whence, from (5)

$$M = -\tfrac{1}{6}W_1 l + \tfrac{1}{6}l\left(\frac{W_1}{2} - \frac{W_2}{2}\right)$$

$$= -\tfrac{1}{12}l(W_1 + W_2) \quad \ldots\ldots\ldots \quad (7)$$

With both bays loaded with $W$ (i.e. two loads of $\frac{W}{2}$ on each), this gives

$$M = -\tfrac{1}{6}Wl \quad \ldots\ldots\ldots \quad (7a)$$

With one bay loaded with $W$ (that is two loads of $\frac{W}{2}$ on it) and the other unloaded, it gives

$$M = -\tfrac{1}{12}Wl \quad \ldots\ldots\ldots \quad (7b)$$

Formula 7*a* is the basis for the construction of bending moment diagrams for dead load, and for live load both spans loaded.

Formula 7*b* is the basis for the construction of bending moment diagrams for live load one bay only loaded.

Consider now the reactions on the supports.

From Fig. 15 (*b*), left-hand half, taking moments about $R_2$

$$R_1 l - \frac{W_1}{2} \cdot \tfrac{2}{3}l - \frac{W_1}{2} \cdot \tfrac{1}{3}l = M$$

or $$R_1 l - \frac{W_1 l}{2} = M$$

whence $$R_1 = \frac{M}{l} + \frac{W_1}{2}$$

similarly $$R_3 = \frac{M}{l} + \frac{W_2}{2}$$

and clearly $$R_1 + R_2 + R_3 = W_1 + W_2$$

$$\therefore\ R_2 = (W_1 + W_2) - R_1 - R_3$$

$$= W_1 + W_2 - \frac{M}{l} - \frac{W_1}{2} - \frac{M}{l} - \frac{W_2}{2}$$

$$= \frac{W_1 + W_2}{2} - \frac{2M}{l}$$

$$= \frac{W_1 + W_2}{2} + \tfrac{1}{6}(W_1 + W_2)$$

$$= \tfrac{2}{3}(W_1 + W_2) \quad \ldots\ldots\ldots \quad (8)$$

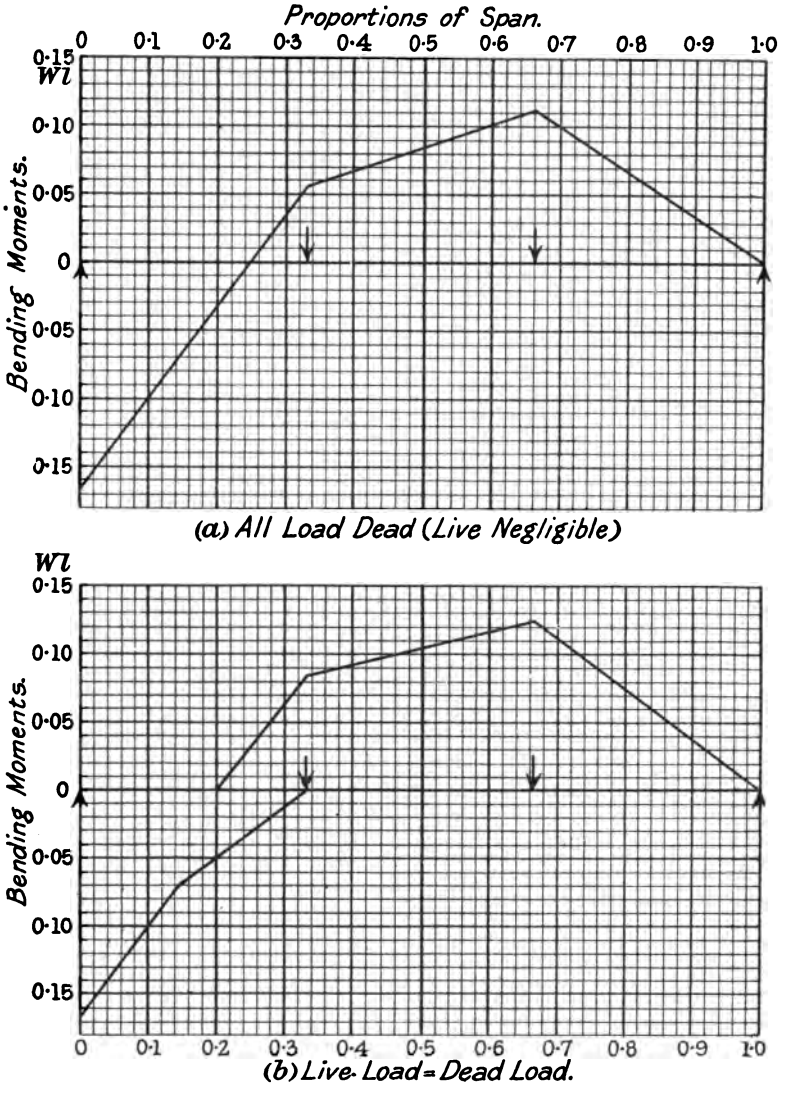



Fig. 17. Curve of Maximum Moments. Two Spans. Two Point Loads at Third Points. Various Ratios of Live Load to Dead Load.

The outer reaction

$$R_1 = \frac{W_1}{2} + \frac{M}{l}$$

$$= \frac{W_1}{2} - \tfrac{1}{12}(W_1 + W_2)$$

$$= \tfrac{5}{12}W_1 - \tfrac{1}{12}W_2 \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Hence, with both spans loaded to $W$,

$$R_1 = \tfrac{1}{3}W \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

and with one span only loaded, the outer reaction

$$R_1 = \tfrac{5}{12}W \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

Note this particularly, that the outer reaction is 25 per cent. greater with one bay only loaded, than when both bays are loaded.

The outer reaction of the unloaded span is

$$R_1 = -\tfrac{1}{12}W \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

The negative sign means that the reaction is upward, producing tension in the column below it if this is the sole restraint.

Selecting the worst cases of these results and expressing as live load and dead load, we have [denoting total load by $W_t$, live load by $W_l$ and dead load by $W_d$, so that $W_t = W_d + W_l$]

Max. moment at centre support

$$M = -\tfrac{1}{6}W_t l = -\tfrac{1}{6}(W_l + W_d)l. \quad . \quad . \quad (13)$$

Max. reaction at centre support

$$R_2 = \tfrac{4}{3}W_t = \tfrac{4}{3}(W_l + W_d) \quad . \quad . \quad . \quad . \quad (14)$$

Max. reaction at outer support

$$R_1 = \tfrac{5}{12}W_t - \tfrac{1}{12}W_d = \tfrac{5}{12}W_l + \tfrac{1}{3}W_d \quad . \quad . \quad (15)$$

Min. positive reaction or max. negative reaction at outer support

$$R_1 = \tfrac{5}{12}W_d - \tfrac{1}{12}W_t = \tfrac{1}{3}W_d - \tfrac{1}{12}W_l \quad . \quad . \quad (16)$$

Referring now to the diagrams, Fig. 16 (*a*) gives the diagram for the dead load (which, of course, acts equally on both spans), and also for the live load in the particular condition when both spans are loaded.

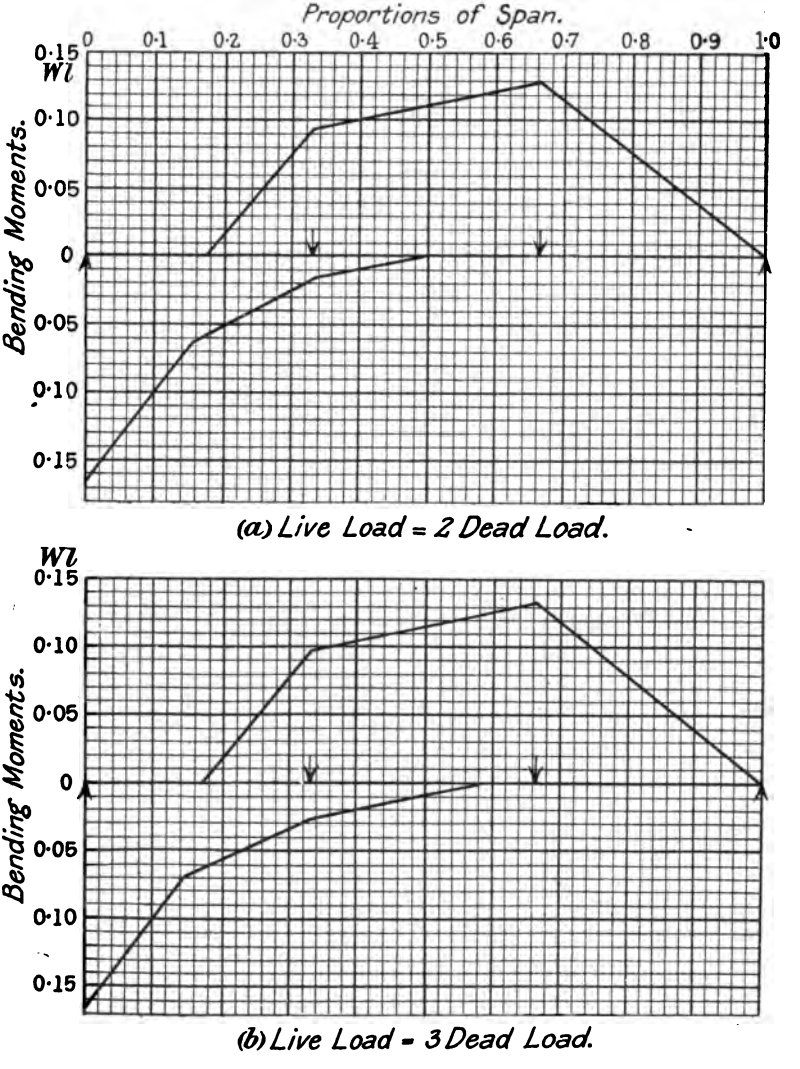



**Fig. 18. Curve of Maximum Moments. Two Spans. Two Point Loads at Third Points. Various Ratios of Live Load to Dead Load.**

The usual bending moment diagram for two point loads in which the moment increases from zero at the two ends to $\frac{Wl}{6}$ at the loads and remains constant at this value between the two loads (i.e. over the centre third) is erected on two sloping base lines drawn from the intersection of the main base line with the outer supports to a point on the central support representing the negative bending moment of $\frac{Wl}{6}$ in accordance with Formula 7 (*a*). Notice that the effect of continuity is to reduce the positive moment from $\frac{Wl}{6}$ to $\frac{Wl}{9}$ at the outer load, $\frac{Wl}{18}$ at the inner load, to produce a bending moment of $-\frac{Wl}{6}$ at the central support, which, with free beams, would be unstressed (except in shear). These reductions in the positive bending moment only occur for the condition of both spans loaded; with one span only loaded the reductions still occur but to a lesser degree.

Fig. 16 (*b*) shows the live load curve when one span only is loaded (the right-hand one). The negative moment at the support is now only $\frac{Wl}{12}$ (see Formula 7 (*b*)), and this provides a basis on which a free bending moment curve is erected on the loaded side while the negative moments on the unloaded side are derived directly by joining this point to the axis of the base line at the outer support. It will be seen that the positive bending moment at the outer load is now reduced from $\frac{Wl}{6}$ to $\frac{5}{36}Wl$ and at the inner load to $\frac{Wl}{9}$, while the fact of continuity has incurred on the designer the liability to provide for negative bending moments varying from $-\frac{Wl}{12}$ at the centre support to zero at the outer supports.

In Fig. 16 (*c*) the above curves are combined, the worst cases, as before, being selected. As before, the live load on the

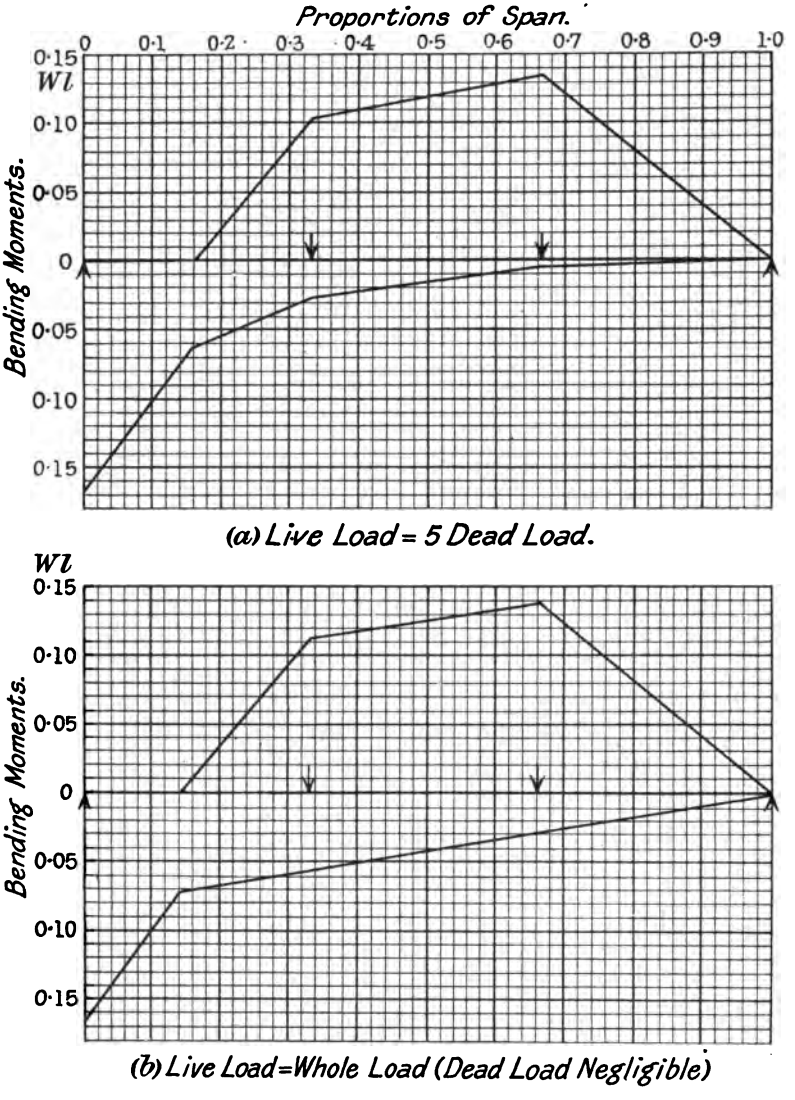



Fig. 19. Curve of Maximum Moments. Two Spans. Two Point Loads at Third Points. Various Ratios of Live Load to Dead Load.

one span condition may apply to either span; hence the symmetry of diagram 16 (*c*) about the central support.

Figs. 17 (*a*) and 19 (*b*) are enlarged diagrams of the right-hand part of 16 (*c*), the former being the dead load curve and the latter the two live load curves. These diagrams are drawn in such a way as to allow of the ready reading off of the values at any distance along the span and they may, if desired, be used for calculating separately the bending moments due to the dead load and live load of any structure subjected, and both of these moments may then be added together with due regard to signs.

As before, it is, however, very much more convenient for curves of combined live and dead loads to be presented for ready reference and the design of the reinforcement in practical beams can be done much more conveniently when these curves are so presented. This is done in Figs. 17 (*b*) to 19 (*a*).

To make it perfectly clear to the student how these curves of combined live load and dead load are arrived at, consider, for example, the beam section at a third of the span from the central support. From Fig. 16 we see that the dead load moment here is

$$\frac{Wl}{18} = \cdot 0555$$

while the positive live load moment at this point is

$$\frac{Wl}{9} = \cdot 1111$$

To find the point on the combined curve for live load equals twice dead load, we notice that one-third of the total load will then be dead and two-thirds of the total load live and the bending moment at this point will therefore be

$$\tfrac{2}{3} \times \cdot 1111 + \tfrac{1}{3} \times \cdot 0555 = \cdot 0926$$

which, it will be noticed, is a point on the curve of Fig. 18 (*a*).

Similarly, the corresponding negative moment at this same beam section is arrived at as follows. The dead load moment from Fig. 16 (*c*) is + ·0555 and the negative live load moment is − ·0555. The combined moment for live load equals twice dead load is therefore

$$+ \cdot 0555 \times \tfrac{1}{3} - \cdot 0555 \times \tfrac{2}{3} = - \cdot 0185$$

These figures are, of course, in terms of the *total* load on the beam.

Thus, to take an actual example, suppose the span is 20 ft., the live point load 10 tons and the dead point load 5 tons, then $W$, the total load on the beam, would be twice times 15 = 30 tons, and the worst bending moment which the beam need be designed for at a point distant one-third of the span from the central support (i.e. at the inside load) is

$$+ \cdot 0926 Wl$$
$$= \cdot 0926 \times 30^T \times 20'$$
$$= 55 \cdot 56 \text{ ft. tons}$$
$$= 1{,}500{,}000 \text{ in. lbs.}$$

Remember that a positive bending moment is a bending moment producing compression at the top of the beam and tension at the bottom, i.e. the kind of bending moment which ordinarily exists in a beam freely supported, loaded near the middle.

The negative bending moment to be provided for at this section is

$$- \cdot 0185 \times 20' \times 30^T$$
$$= 11 \cdot 11 \text{ ft. tons}$$
$$= 300{,}000 \text{ in. lbs.}$$

Remember that a negative bending moment is one producing tension at the top of the beam and compression at the bottom, so that this moment enables us to determine how much steel at this point is necessary along the top of the beam to take the tension and to determine whether the compressive stresses produced by this negative moment at the bottom of the beam is safe.

The diagram shows that at this particular point five times as much steel is necessary at the bottom of the beam as at the top from considerations of carrying the necessary tension to resist these bending moments.

Examples of the use of these curves are given later when dealing with actual examples of design.

**Reactions on Supports.** Notice particularly the results of Formulae (8)–(12) and (14)–(16).

With beams free at the ends and loaded symmetrically, the reactions would be half the load at each end, which would give

a reaction at the outer support of $\frac{W}{2}$, and a reaction at the centre support of $W$. The effect of continuity at the centre support is to lift part of the load off the outer supports and to put it on to the centre one. Thus, when both spans are loaded, the reaction on the outer supports is now only $\frac{W}{3}$ instead of $\frac{W}{2}$, and the reaction on the central support is now $\frac{2}{3}W$ instead of $W$. In other words the beam has lifted one-third of the reaction off the two outer supports and put it on to the centre one. This means that the centre supports, which may be a row of columns, have got to be designed for 33 per cent. more load than if no continuity existed, a very important result which may not be neglected.

In designing the outer supports it is, however, not safe to design them for $\frac{W}{3}$ since, though this represents accurately the reaction when both bays are loaded, a worse reaction is obtained when one bay only is loaded, the value in this case being $\frac{5}{12}W$ in respect of the live load. Since clearly the dead load will always be on both panels, the maximum reaction at the outer support will be $R_1 = \frac{5}{12}W_l + \frac{1}{3}W_d$ [see Formula (15)], and the outer columns or supports should be designed for this reaction.

In cases where the live load is very great as compared to the dead load, the unloaded span will have a tendency to lift off its support and to produce an upward reaction instead of a downward one, the greatest magnitude of this value being given by Formula (16), and this should be allowed for when designing columns. It only occurs, however, when the live load is greater than four times the dead load.

The reader is referred to the end of Chapter II for the treatment of negative moments at end support where this is monolithic.

# CHAPTER V

## A CONTINUOUS BEAM OF MANY SPANS, FREE AT THE OUTER SUPPORTS, WITH UNIFORMLY DISTRIBUTED LOADING

In the preceding chapters the bending moment at any point was perfectly definite on the assumptions made, which included the following:

(*a*) That no settlement of the supports occurred.

(*b*) That the moment of inertia of the beam should be constant for the whole length of the beam and the same in both spans.

(*c*) That the outer ends were so supported as not to develop bending moments at the supports.

(*d*) That the restraint on the beam owing to the central column could be neglected.

(*e*) That the stresses produced would everywhere lie within the elastic limit.

Dealing with more than two spans the problem is just as definite when these conditions hold, but for it to be quite definite we must consider a definite number of spans, i.e. 3 or 4 or 5. When this is done we discover that the moments in the end span do not differ very greatly whether we consider 3, 4, or a greater number of spans and the moments in the interior spans likewise do not differ greatly for a different number of spans. In practice it is sufficiently accurate to use one set of curves and formulae for all interior spans and another set of curves and formulae for all end spans without reference to the number of spans in the beam, and obviously a tremendous amount of duplication and complication is avoided by so doing.

**Interior Spans.** Consider now a continuous beam having a large number of spans, so large that the effect of the end supports in either direction is so small as to be negligible, and consider the spans lying towards the centre of this long beam. These spans were thoroughly investigated in *Reinforced Concrete Design*, Vol. I, Appendix I, 10, and onwards, and very simple

formulae were derived which were also given in *Reinforced Concrete Design*, Vol. I, pp. 165–171. These formulae were adopted by the compilers of the L.C.C. regulations for reinforced concrete and they lead to very simple and convenient results. If a student wishes to follow out their derivation he cannot do better than study Appendix I of *Reinforced Concrete Design*, Vol. I.

As before, the chief difficulty is to calculate the negative moments at the supports and when this has been done the rest is plain sailing. When dealing with a beam of two spans, only three possible conditions can occur*, both bays loaded, one bay loaded, and no bays loaded, and the latter of course may be neglected in design. With a beam of many spans there are of course a very large number of possible conditions and what we have to design for is the worst combination of these.

It may be taken that all these conditions have been analysed and that the worst are the following:

The maximum bending moment at mid-span is produced when alternate bays are loaded and the intermediate bays unloaded (Fig. 20 (*b*)). When this is so it is shown in Vol. I, Appendix I, 10, p. 284, that the bending moment at mid-span is

$$\frac{W_1 l}{12} - \frac{W_2 l}{24}$$

where $W_1$ equals the load on the loaded panels and $W_2$ the load on the unloaded panels. If we consider the live load only, we may put $W_2 = 0$ and the bending moment at mid-span on the loaded panels is then $\frac{Wl}{12}$. If the beam had been free the bending moment would, of course, have been $\frac{Wl}{8}$, and therefore the negative moment at the end which produces this reduction must equal the difference, namely, $\frac{Wl}{24}$.

It will be clear from the symmetry of Fig. 20 (*b*) that the negative moment at all supports will be the same till we get near the end spans of the beam. Fig. 20 (*b*) is therefore constructed by drawing the line of negative moments at a distance

* Unless we consider parts of different spans loaded, which does not give worse and therefore important results.

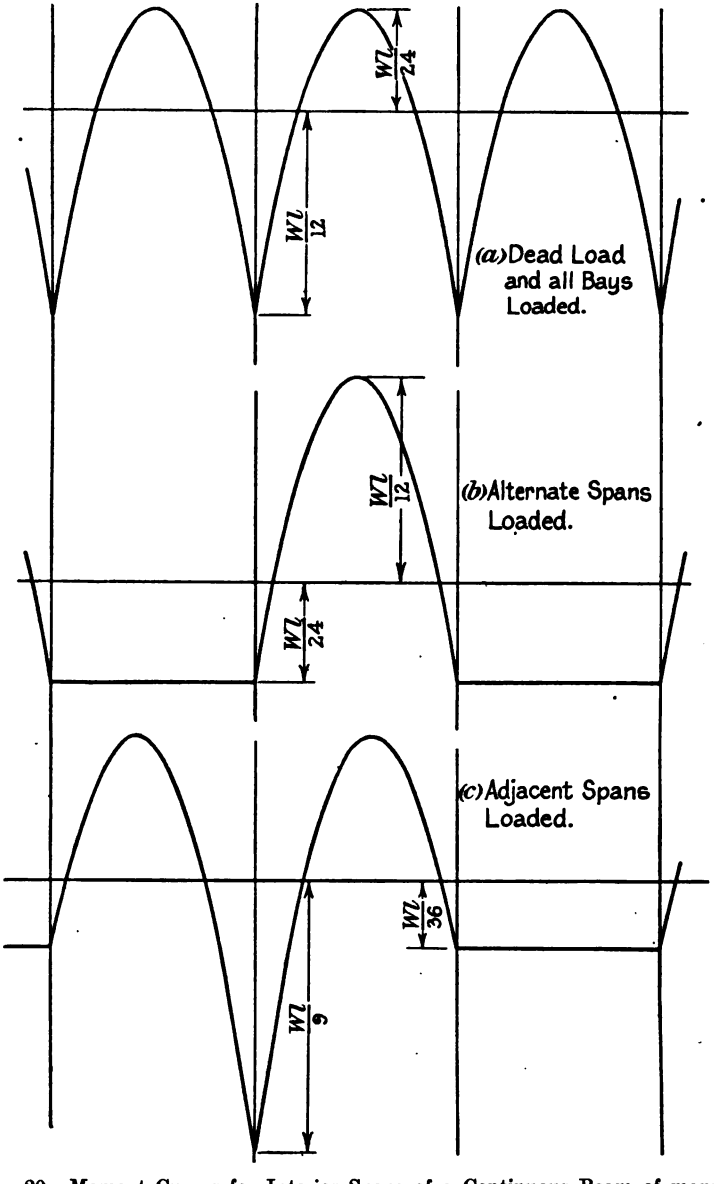



Fig. 20. Moment Curves for Interior Spans of a Continuous Beam of many Spans.

$\frac{Wl}{24}$ below the base line of the diagram and erecting on alternate panels a parabola of height $\frac{Wl}{8}$, producing at mid-spans on alternate spans a positive moment of $\frac{Wl}{12}$ in accordance with the results of the analysis in the Appendix. This gives the worst positive bending moment which any beam will be subjected to.

The worst condition for the bending moment at the supports is given when two adjacent bays are loaded, the bay on either side of these two unloaded, the next two bays loaded, and so on. This produces a negative moment at the support between two loaded bays of $-\frac{Wl}{9}$ and is the worst moment which can be produced by any arrangement of loaded or unloaded panels with a uniform load. The derivation of these formulae is given in Vol. I, Appendix I, p. 285, where it is shown that

$$M_4 = -\frac{W_1 l}{9} + \frac{W_2 l}{36}$$

In this analysis $W_2$ was the load on the least loaded panels and therefore for live load conditions we may put $W_2 = 0$, giving us

$$M_4 = -\frac{Wl}{9}$$

the figure referred to above.

The conditions at the supports are now by no means the same and before we can construct a diagram we also need to know the moment at the support between the loaded and unloaded spans. This is not given but formulae are given from which it may easily be derived.

Thus, from *Reinforced Concrete Design*, Vol. I, Appendix I, 11, p. 285, we have

$$M_2 = -\frac{w_2 l^2}{12} + 2EBa_2 \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Now $w_2 = 0$

$$\therefore M = 2EBa_2$$

$$= -\frac{l^2}{12} \cdot \frac{2B(w_1 - w_2)}{KC + 6B}$$

UNIVERSITY OF CALIFORNIA
DEPARTMENT OF CIVIL ENGINEERING
BERKELEY, CALIFORNIA

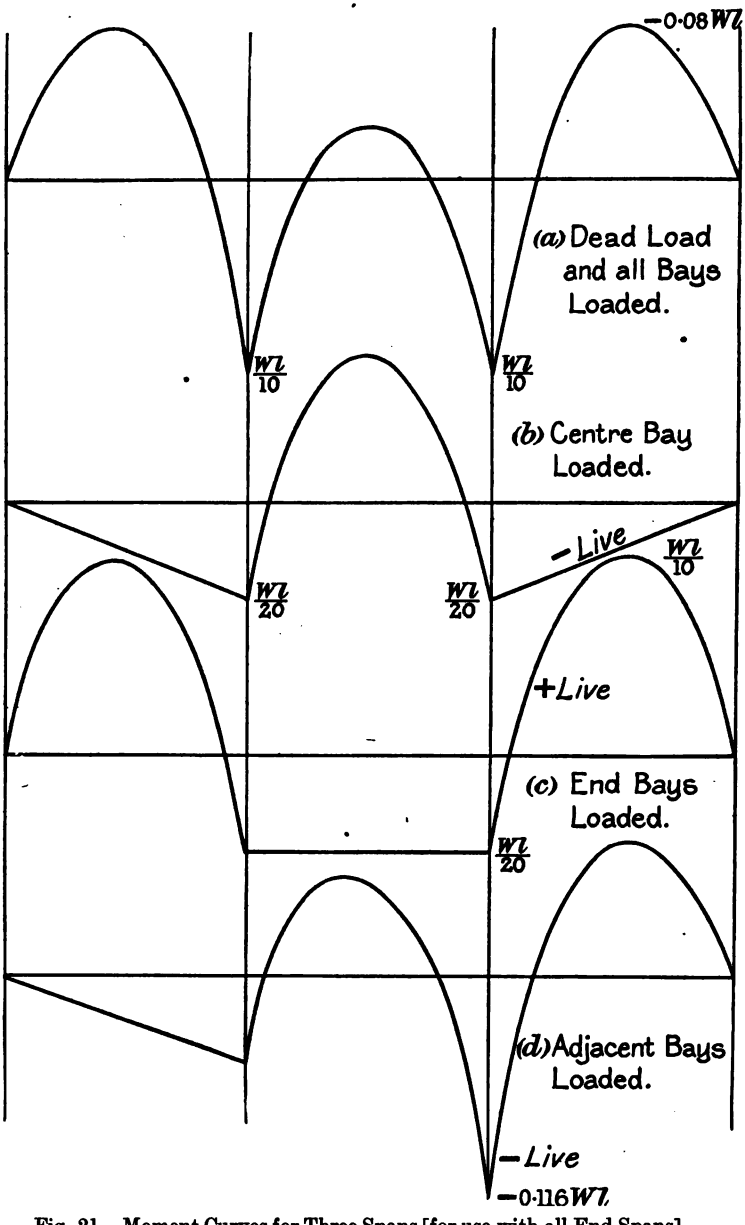



Fig. 21. Moment Curves for Three Spans [for use with all End Spans].

Now, as we are dealing with live load only, $w_1 - w_2 = w$ where $w$ = live load.

We are also neglecting the stiffness of columns, so we may put $KC = 0$, and with these simplifications

$$M_2 = \frac{wl^2}{36}$$
$$= \frac{Wl}{36}$$

where $W$ = live load only.

This calculation shows that the negative moment at the support between the loaded and unloaded spans is $-\frac{Wl}{36}$.

The bending moment diagram in Fig. 20 (*c*) is therefore drawn as follows. Set down $-\frac{Wl}{9}$ at one support and $-\frac{Wl}{36}$ at the two adjacent ones and join these by two sloping lines. On these sloping lines as bases erect skew parabolas of the bending moment as if the beams were free, giving a maximum ordinate of $\frac{Wl}{8}$ at mid-span.

The resultant diagram will be the positive and negative moments about the original base line.

In the unloaded panels there is clearly no parabola to raise above the base so that the negative moment remains $-\frac{Wl}{36}$ throughout the panel.

The dead load diagram is drawn in Fig. 20 (*a*). Clearly the dead load exists equally on all panels and the diagram may therefore be constructed as follows.

In Vol. I, Appendix I, 10, p. 284, we saw that the centre bending moment was

$$M = \frac{W_1 l}{12} - \frac{W_2 l}{24}$$

where $W_1$ and $W_2$ were the loading on alternate panels. For dead load clearly $W_1 = W_2$ and then

$$M = \frac{Wl}{24}$$

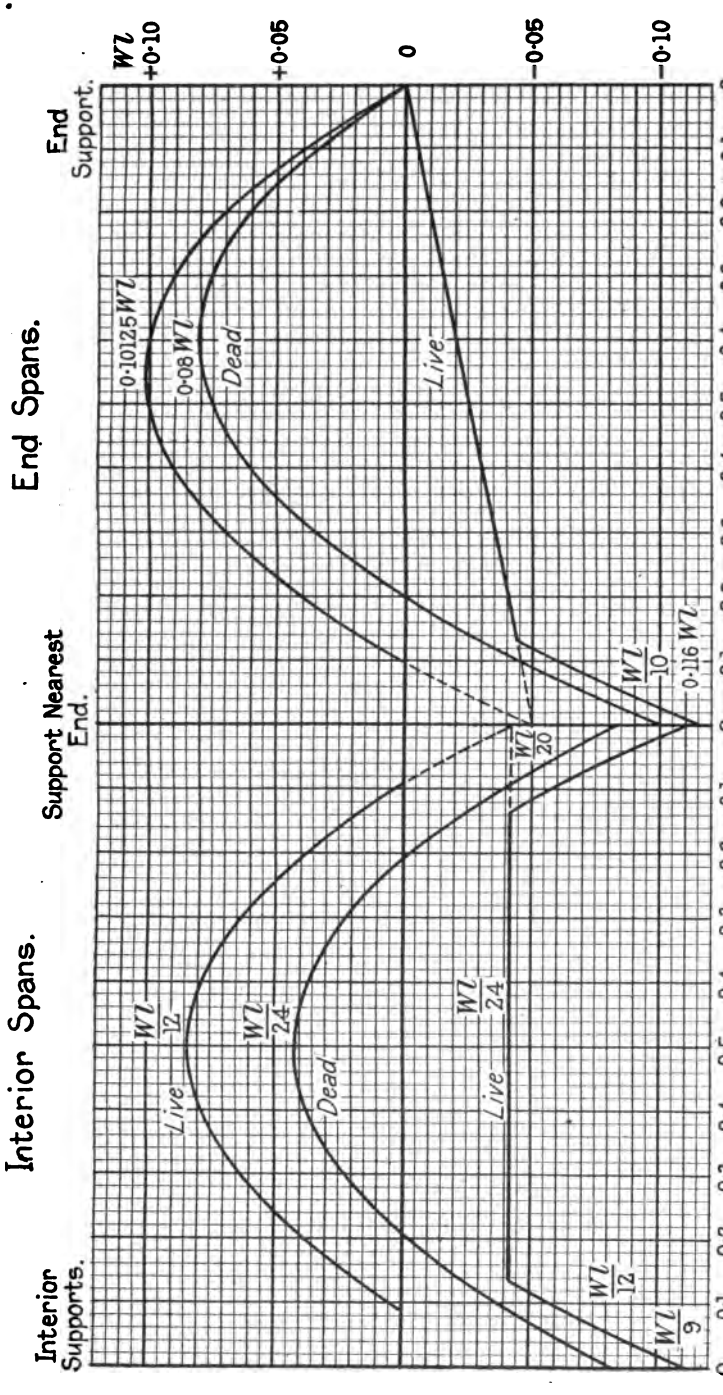



Fig. 22. Curve of Maximum Moments. Interior and End Spans. Uniform Load Distribution.

It follows that with the positive moment at mid-span reduced from $\frac{Wl}{8}$ when the beams are free to $\frac{Wl}{24}$ for continuous beams, this implies a negative moment of the difference, i.e. $-\frac{Wl}{12}$ at the support.

Fig. 20 (*a*) is therefore drawn by setting down $\frac{Wl}{12}$ from the base line at each support, joining these points up by a fresh horizontal base line parallel to the first, and erecting on this in each span a bending moment diagram as if the beam were free, i.e. a parabola having a maximum ordinate at mid-span of $\frac{Wl}{8}$. This gives a dead load bending moment curve with a maximum value of $+\frac{Wl}{24}$ at mid-span and $-\frac{Wl}{12}$ at the support. The left-hand half of Fig. 22 is compiled from Fig. 20 (*a*) to (*c*), so as to give the worst moments at any particular section for all cases.

The maximum positive live load is derived from Fig. 20 (*b*), and the upper portion of this curve is therefore used.

The worst condition for the negative moment near the middle of the beam is given by the unloaded panel in Fig. 20 (*b*) which is therefore also used.

The worst case near the support is given by the bottom cusp of Fig. 20 (*c*), and these two last curves combined give the curve of maximum negative moments due to live load.

The dead load curve from Fig. 20 (*a*) is shown dotted and is included although it lies inside the other curves, since in any practical case part of the load is dead and it is therefore more economical to apply the smaller dead load bending moments, and to confine the high values of the live load bending moments to the live load.

**End Bays.** As was previously explained, the moments in the end spans differ very little whether we are considering the end spans of a series of 3, 4, 5 or more spans, and we may therefore adopt the simpler case of 3 spans in our investigation of end spans.

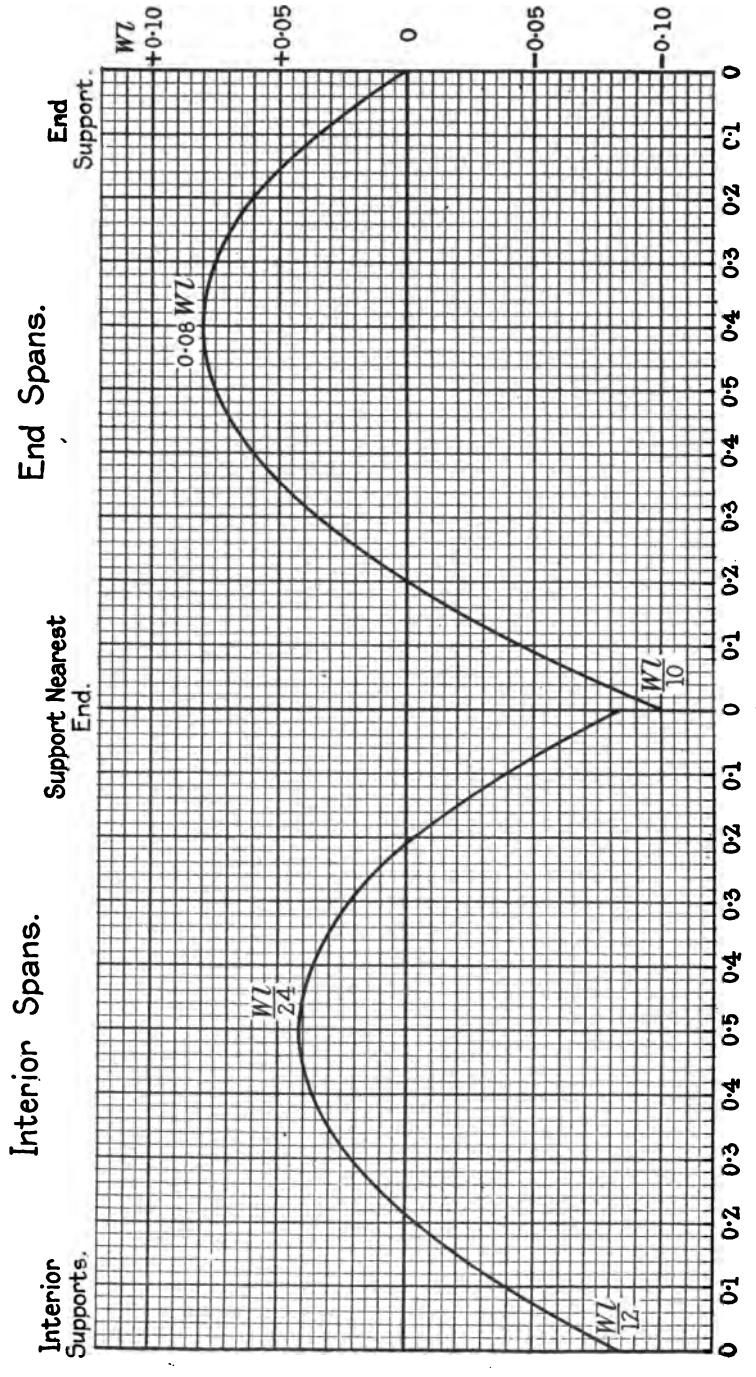



Fig. 23. Curve of Maximum Moments. Interior and End Spans. Uniform Load, all dead [Live Load negligible].

Exactly similar investigation to the last enables us to draw bending moment curves for three spans. We have to consider the following cases:

The dead load curve as in Fig. 21 (*a*) when all spans are loaded.

The live load positive curve, for which the worst case in the end spans is when the outer spans are loaded, and the centre span unloaded (Fig. 21 (*c*)).

The worst case for the negative live load moment at the support inside the end span is given when two adjacent spans are loaded and the third one unloaded, as in Fig. 21 (*d*).

The worst case for the negative live load moment towards the middle of the end span is given when the end spans are unloaded, and the centre one loaded, as shown on Fig. 21 (*b*).

The values of the moments at the supports are easily obtained by the theorem of three moments, given later, or by simplifying from formulae in Appendix I of Vol. I, and the parabolas on the new base lines are then drawn as before, and need not be further explained.

Note in all these diagrams the bending moment on the outermost support is brought down to zero, since it will be remembered that this is the support where the beam is assumed free and not restrained.

We must now consider whether we are justified in adopting the moment curve for the end span of three spans for all end spans, and we will consider the negative moment at the support inside the end panel.

The results of rigid analysis by the theorem of three moments or otherwise gives the following results:

| | Negative moment at support inside end panel |
|---|---|
| Number of spans | Dead load curve |
| 3 | ·10 *Wl* |
| 4 | ·107 *Wl* |
| 5 | ·105 *Wl* |

Now, all these figures were derived on the assumption that the moment of inertia is the same in the beam for the outer span as it is for the interior spans. When we were considering the case of two spans this was true, because, of course, both

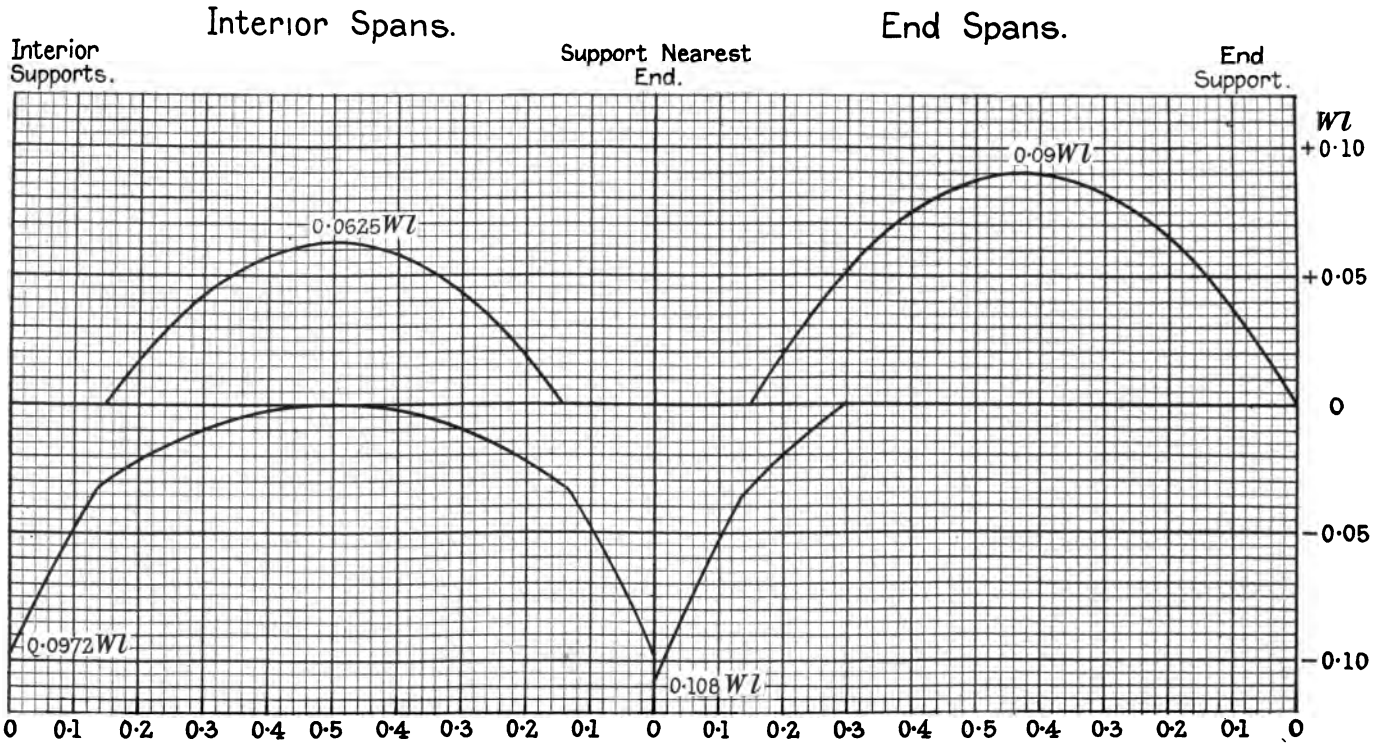



Fig. 24. Curve of Maximum Moments. Interior and End Spans. Uniform Load. Live Load = Dead Load.

spans were outer spans and both spans were designed for the same bending moments, but when we are dealing with three or four spans we shall find that the end span has to be designed for considerably greater moments for the whole of its middle portion than do the interior spans, and the effect of this is that the moment of inertia of the end spans is always considerably greater than for the interior spans.

The effect of this is to increase the bending moment which is resisted as a positive moment near the middle and to reduce the bending moment produced at the support.

This question was investigated in Vol. I, Appendix I, 8, p. 279, when the moment of inertia of the outer beam of three spans was made one and a quarter as great as that of the interior span. The case investigated was a somewhat difficult one in which the stiffness and strength of the columns was taken into account in determining the bending moments in the beams, but the formulae in this Appendix may be simplified by writing $KC$ = zero throughout, which eliminates the effect of the columns, and it will then be seen that the effect of the end spans having a greater moment of inertia has been to reduce the moment on the interior support.

It is for this reason that the author prefers to use the figures for three spans as his standard for all spans sooner than one of the others, because, though the results differ very slightly, the effect of greater stiffness of the end spans is to give a lesser moment at the support and a greater one at mid-span, which we do by adopting the lower figure of $\cdot 10Wl$ at the inside support sooner than one of the higher figures.

This also makes the author's curves agree with L.C.C. regulations which give $\frac{Wl}{10}$ as the dead load moment at the support one span from the end support.

In the case of the live load, which after all is generally by far the greater portion of the load, the agreement between the results obtained by exact calculation for different numbers of spans is even greater. Thus the negative moments at the support inside the end span, due to the worst *live* load conditions, calculated in the ordinary way, give the following results:

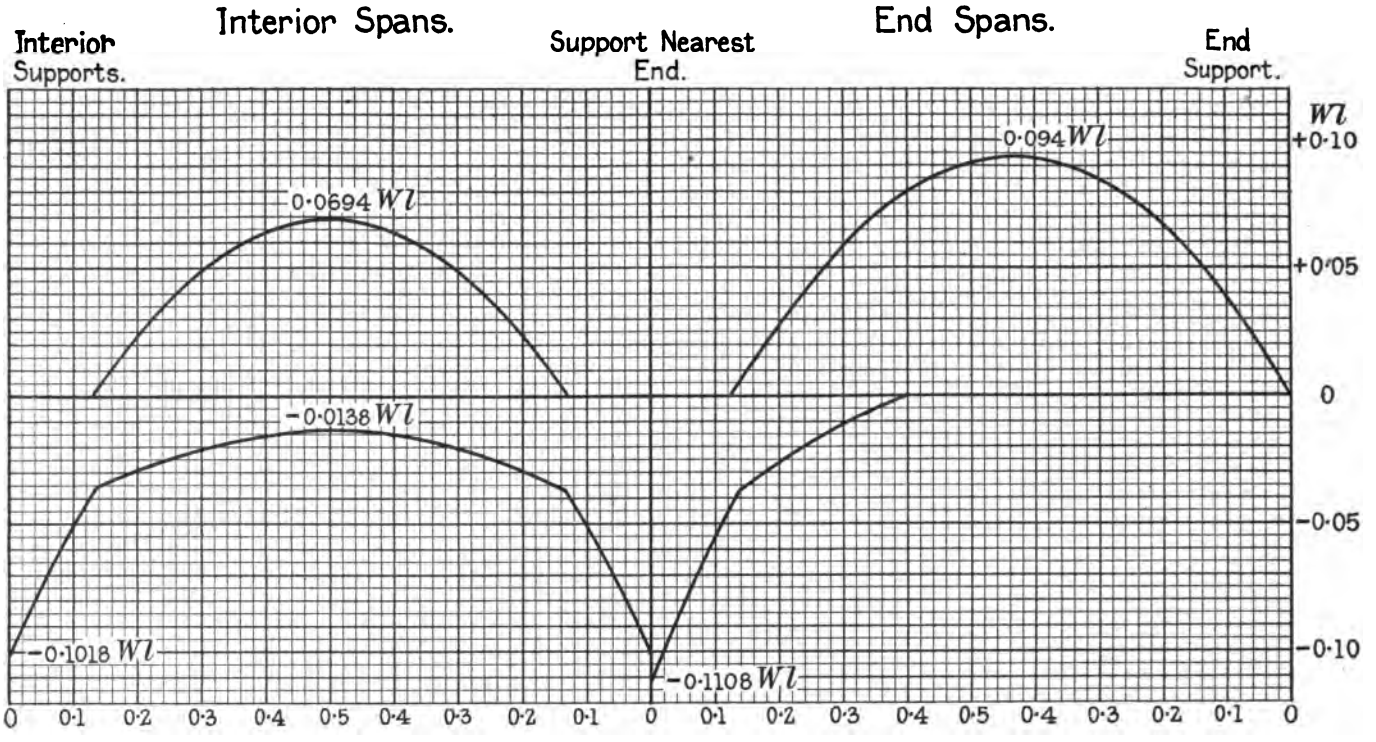



**Fig. 25. Curve of Maximum Moments. Interior and End Spans. Uniform Load. Live Load = 2 Dead Load.**

| Number of spans | Negative moment at support inside end span<br>Live load |
|---|---|
| 3 | ·116*Wl* |
| 4 | ·120 |
| 5 | ·119 |

The value which we have taken by adopting the end bay of three spans as our standard is therefore ·116*Wl*.

With these comments we may now adopt the worst conditions of the end span from Fig. 21 and use them in our curve of maximum moments for *all* end spans, on the right-hand half of Fig. 22.

It will be noticed that the maximum positive moments are considerably greater in the end panel than for the interior ones, while the negative moments at mid-span are somewhat less.

Figs. 23 to 28 give curves of the maximum bending moments for different ratios of live load to dead load calculated by combining in definite proportions the live load moment and dead load moment for each point, exactly as was described in previous chapters.

The usefulness of these curves in everyday practice is very great where a saving of time and accuracy are both required, since they enable us to tell at a glance not only what the maximum bending moments are at the centre supports which need to be designed for, which could be arrived at more simply, but also where the various bars may be permitted to leave the top and bottom boom to be bent up for shear or may be stopped off. They show, for example, how far along the span the negative moment producing tension at the top can occur and what the magnitude of the reinforcement necessary at different points requires to be.

Coming now to the question of the reaction of supports, it will be remembered that in the case of the two span beams it was pointed out that the effect of continuity was to take a certain load from the outermost support and transfer it to the interior supports. Something of the same sort occurs at the end panels of continuous beams of several spans but to a lesser extent. This subject was investigated in Vol. I, p. 117 and onwards, and it was shown that the maximum increase resulting from this cause was 10 per cent. in the case of three spans fully

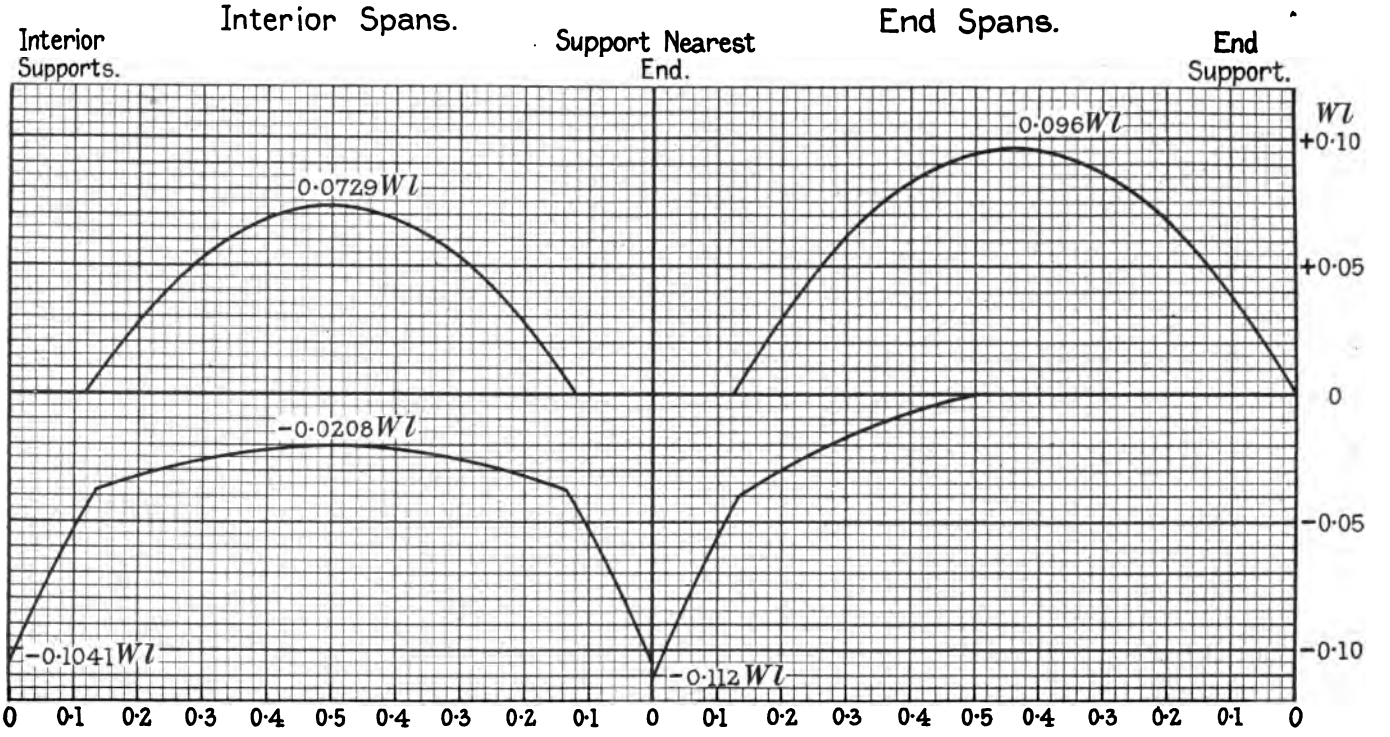



**Fig. 26. Curve of Maximum Moments. Interior and End Spans. Uniform Load. Live Load = 3 Dead Load.**

loaded, but might reach the value of 20 per cent. when two were loaded and the remaining one unloaded. This of course applies to the live load only, and for dead load the increase is only 10 per cent. as above. Hence when live load equals dead load it is desirable to allow for an increase of 15 per cent., when live load equals twice dead load to allow for 16·6 per cent., and so on.

It should be noted that placing an interior span moment curve and an end span moment curve adjacent, as we do in Fig. 21, has the effect that the dead load curves do not have the same value at the support where they meet. The same applies to the live load curves to a lesser degree.

The higher values on the right-hand side are to be considered as applicable to *both* sides of the support one span from the end, except in cases where the small moment represented by the difference in value of the moment on the two sides of this support may safely be taken by the column. The left-hand diagram, of course, is strictly that for a span well removed from the end, but it is quite accurate enough to use it for all spans but the end one as we have decided to do, and very great convenience results. This treatment also agrees with L.C.C. regulations.

The reader is referred to the end of Chapter II for the treatment of negative moments at end supports where some restraint is afforded.

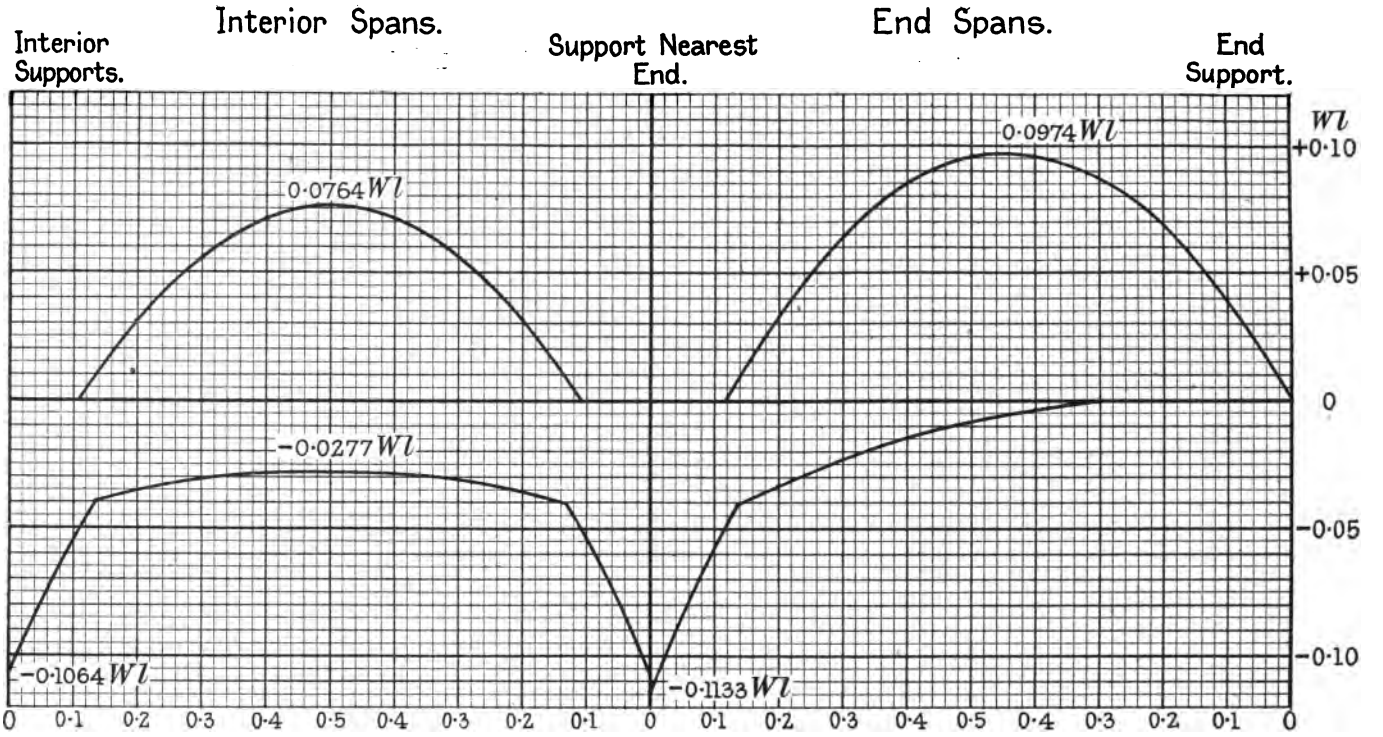



Fig. 27. Curve of Maximum Moments. Interior and End Spans. Uniform Load. Live Load = 5 Dead Load.

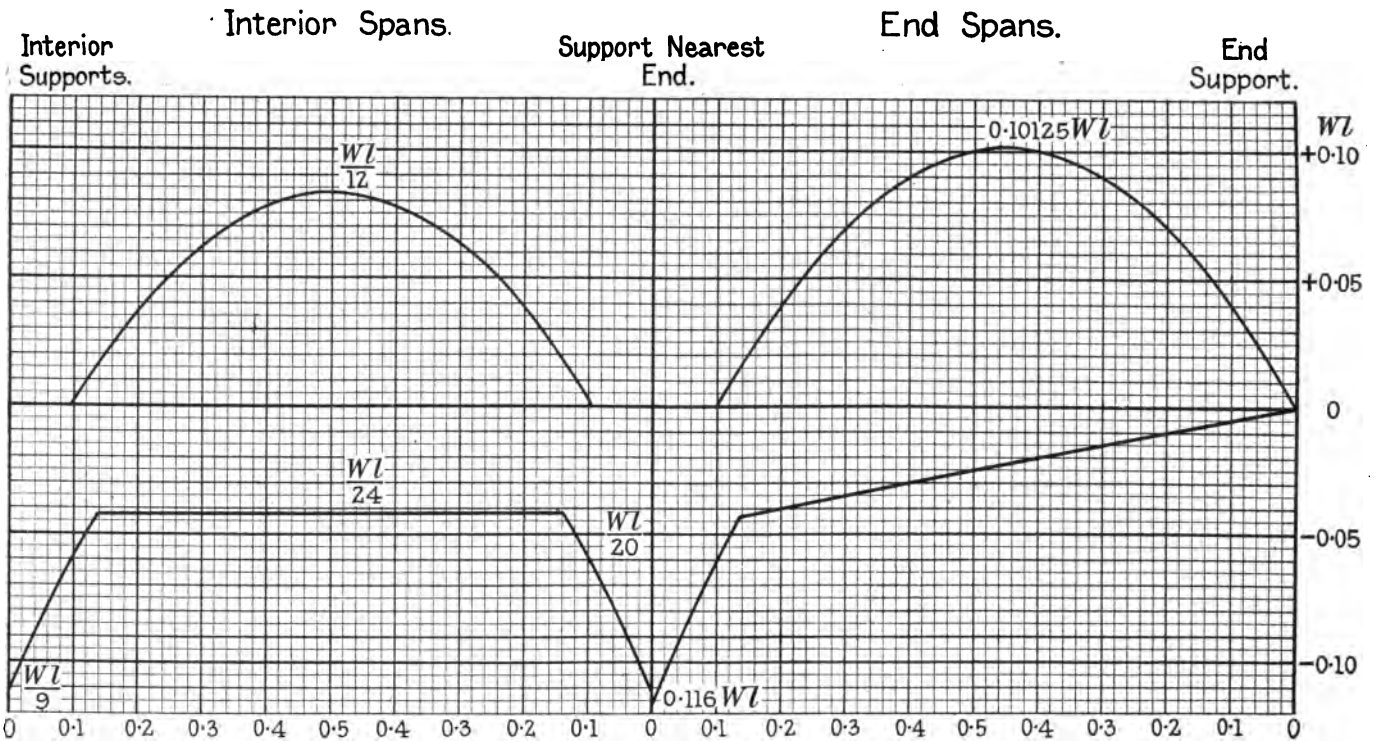



**Fig. 28. Curve of Maximum Moments. Interior and End Spans. Uniform Loads. All Load Live [Dead Load negligible].**

## CHAPTER VI

### A CONTINUOUS BEAM OF MANY SPANS, LOADED WITH POINT LOADS AT MID-SPAN

The explanation given in the previous chapter as to how these curves are derived applies very largely to the present case and therefore the explanation may be less full.

For the interior spans the value of the bending moment at mid-span was calculated in Vol. I, Appendix I, 14, pp. 288, 289, both for live and dead loads and the construction follows from these as described in the preceding chapter.

The moments in the end spans are derived by considering the case of three spans, for the same reasons as were given in the preceding chapter.

With the explanation given in the previous chapters, no difficulty will be found in following the significance or the construction of curves in Figs. 29 and 30, the former applying to interior and the latter to three spans.

Fig. 31 shows these curves combined to form curves of maximum moments at any section exactly as was done previously, and Figs. 32 to 37 are curves in which live load and dead load have been combined in different ratios so as to give the maximum positive and negative moments at any section for different ratios of live load to dead load by simple reference and without further calculation.

Some difficulty may be felt in the idea of having the dead load as a point load concentrated at the centre and it may be asked whether this would not in fact really be a distributed load. Reference to Fig. 8, showing the arrangement of beams in a panel which most frequently gives in practical application of the case which we are investigating, will show that any particular panel which happens to be unloaded will still have the dead load of the weight of the floor slab transmitted to the secondary beam, and the secondary beam will still transmit this load, together with its own weight, and will apply this as a

concentrated load to the centre of the main beam, and the whole of its dead load therefore acts as a concentrated load on the main beam.

The only portion of the dead load which would not so act is the weight of the main beam itself. This is generally so small

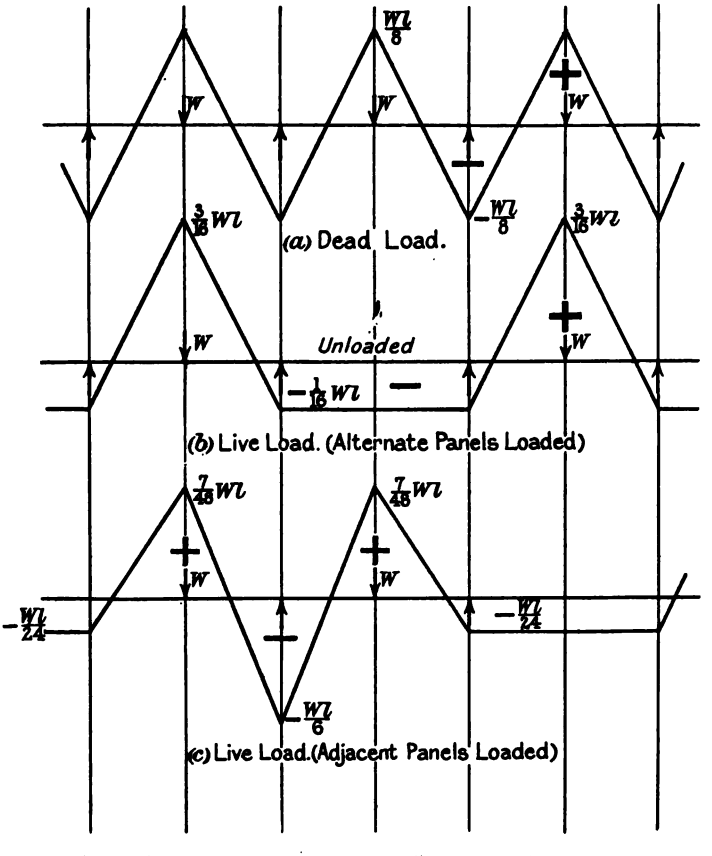



Fig. 29. Curve of Moments. Interior Spans. Concentrated Loads at Mid-span.

compared with the dead load of the floor slab and secondary beams that, without appreciable error, it may be included in it, but if in any particular case it is desired to separate them, this may, of course, be done by superimposing the dead load bending moments arrived at in the previous chapter, as calculated for

the dead load of the main beam only, to the moments arrived at for the rest of the loads by the curves of this chapter.

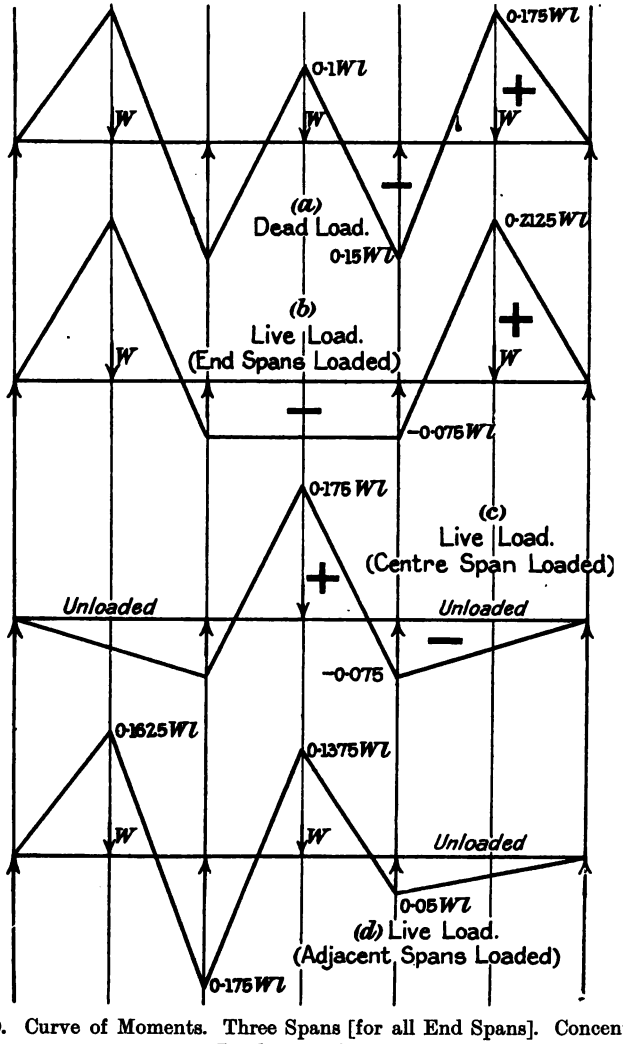



Fig. 30. Curve of Moments. Three Spans [for all End Spans]. Concentrated Loads at Mid-span.

The reader is referred to the end of Chapter II for the treatment of negative moments at end supports where some restraint exists.

UNIV. OF CALIFORNIA

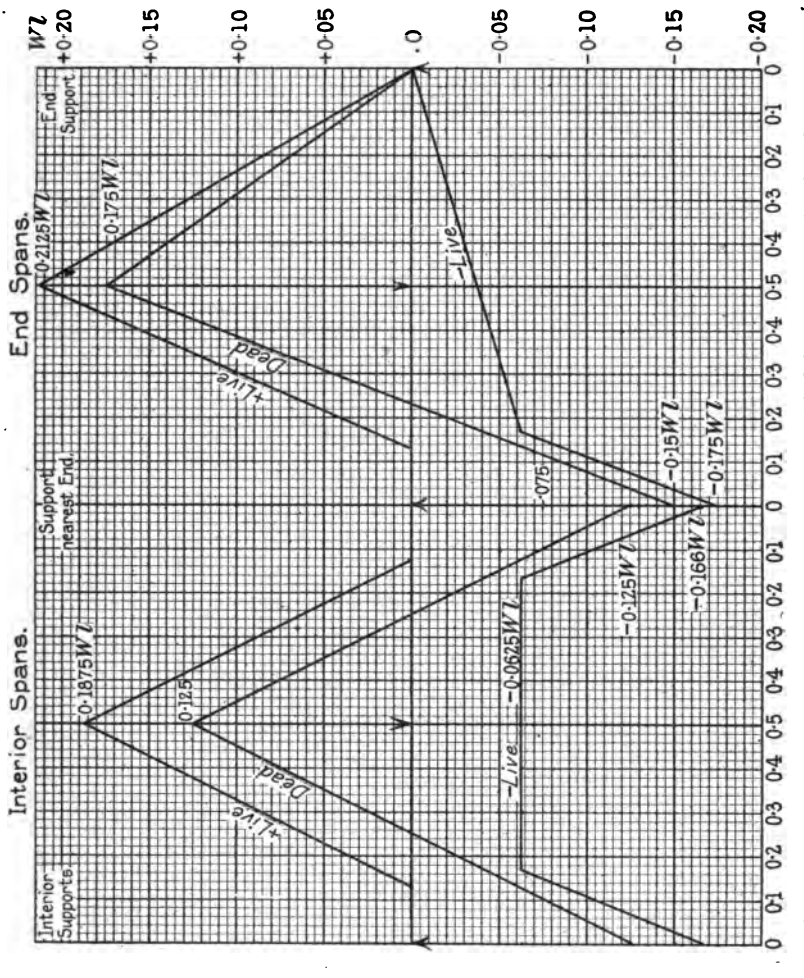



Fig. 31. Curve of Maximum Moments. Interior and End Spans. Point Load at Mid-span.

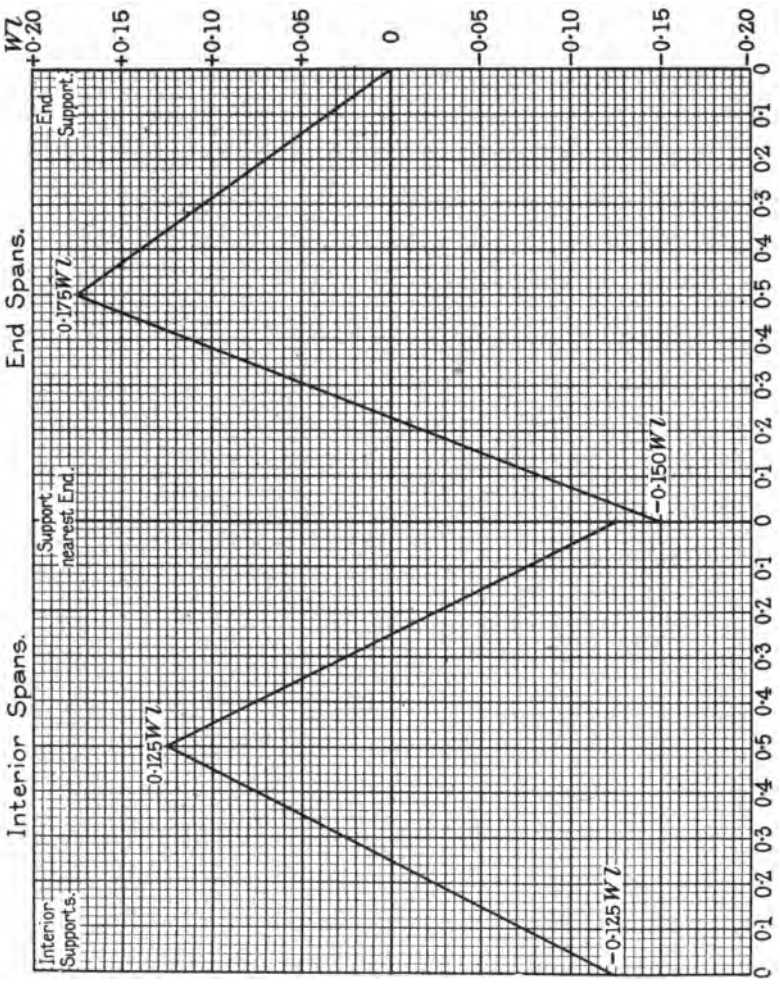



Fig. 32. Curve of Maximum Moments. Interior and End Spans. Concentrated Load at Mid-span. All Load Dead.

UNIV. CALIFORNIA

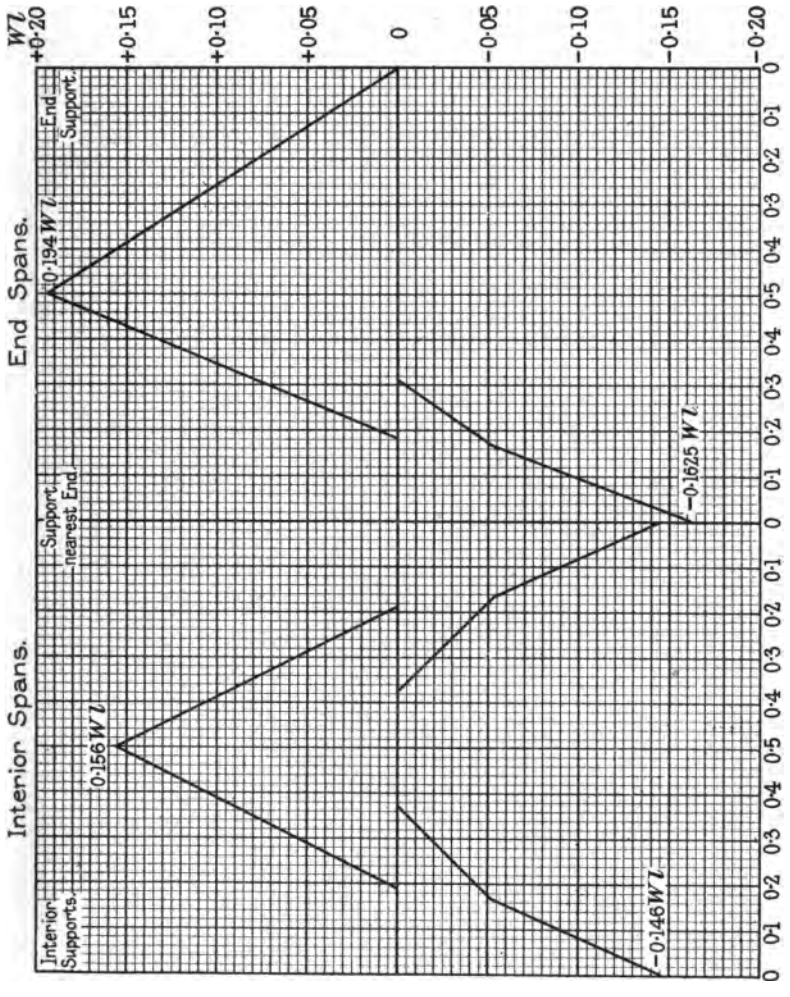



Fig. 33. Curve of Maximum Moments. Interior and End Spans. Concentrated Load at Mid-span. Live Load = Dead Load.

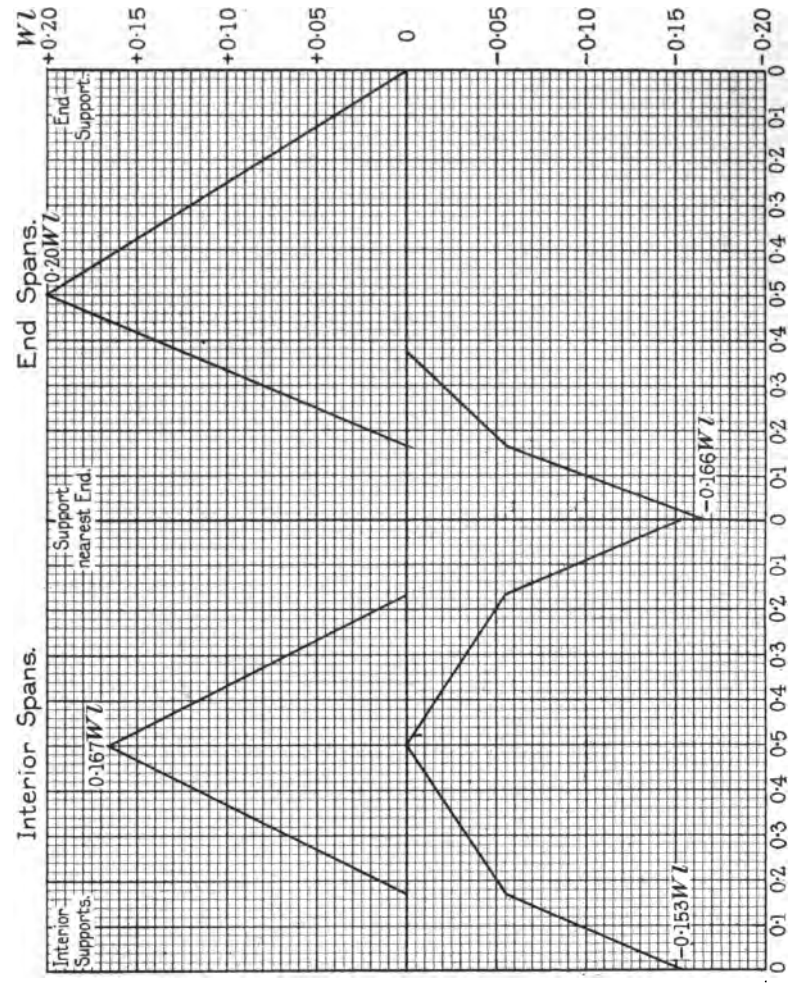



Fig. 34. Curve of Maximum Moments. Interior and End Spans. Concentrated Load at Mid-span. Live Load = 2 Dead Load.

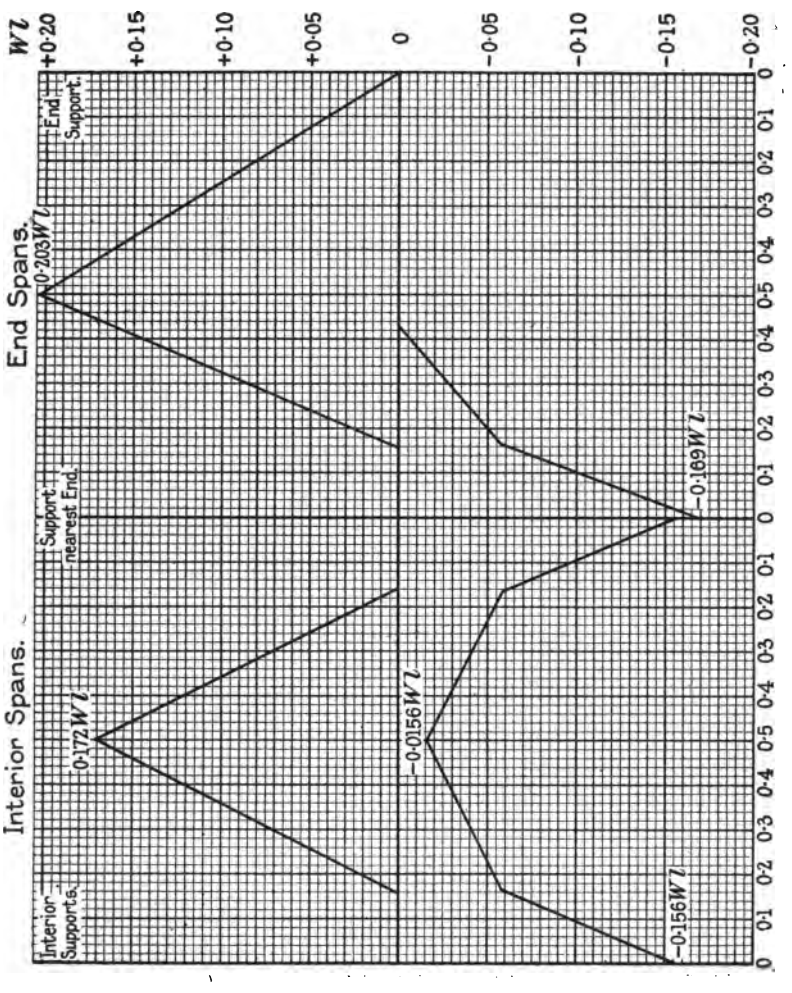



Fig. 35. Curve of Maximum Moments. Interior and End Spans. Concentrated Load at Mid-span. Live Load = 3 Dead Load.

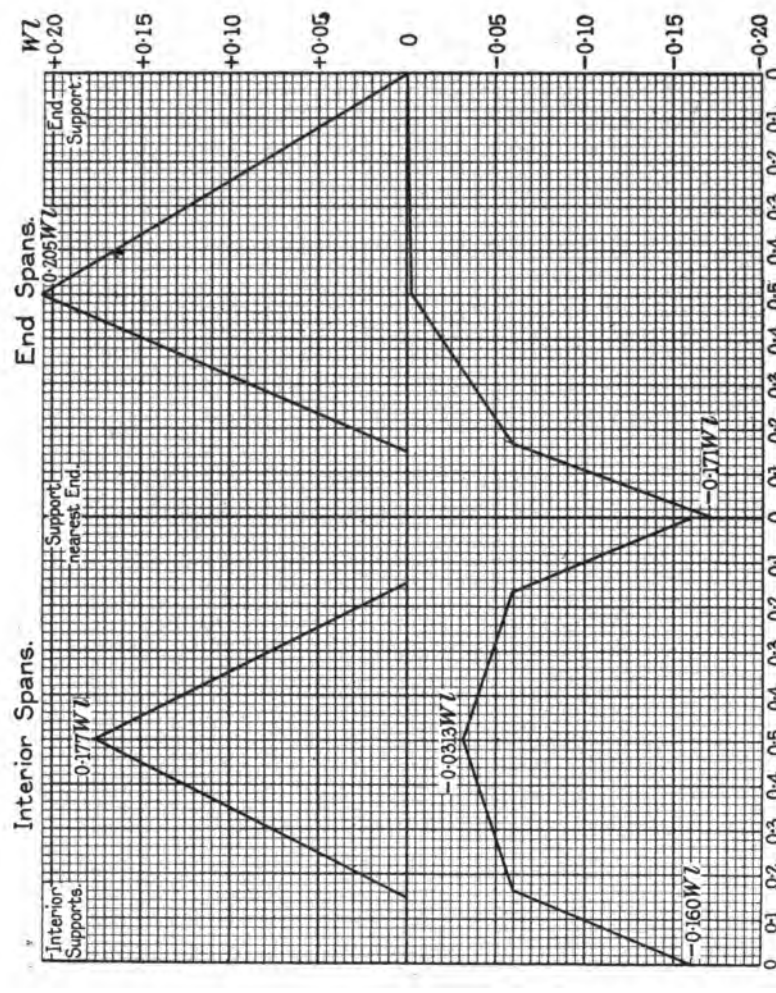



Fig. 36. Curve of Maximum Moments. Interior and End Spans. Concentrated Load at Mid-span. Live Load = 5 Dead Load.

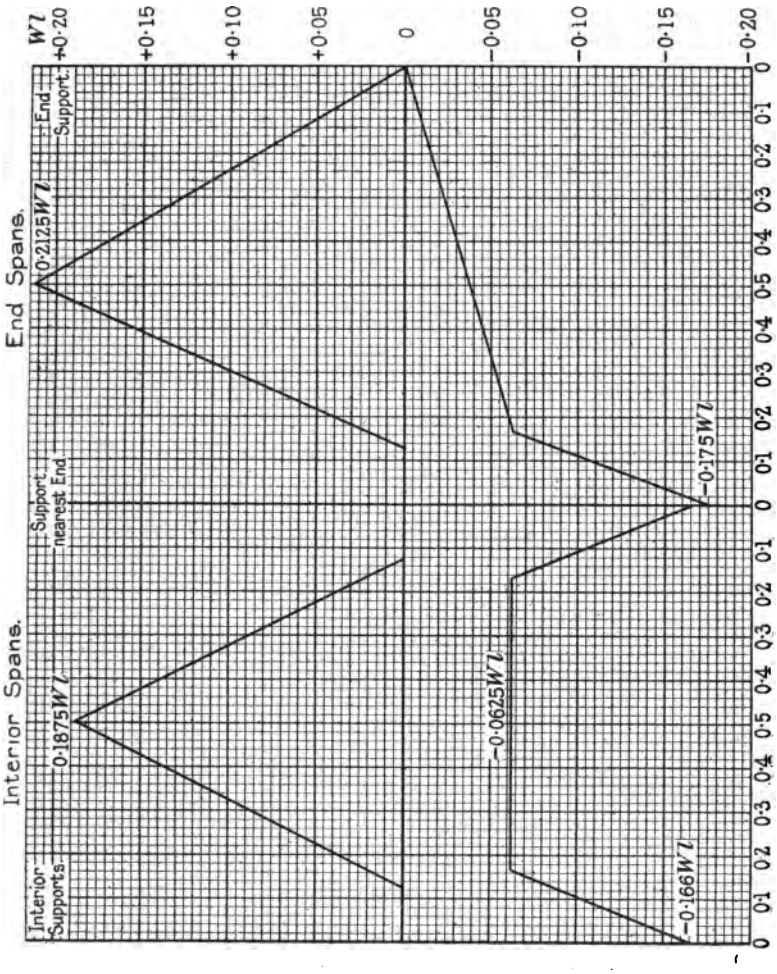



Fig. 37. Curve of Maximum Moments. Interior and End Spans. Concentrated Load at Mid-span. All Load Live [Dead Load negligible].

## CHAPTER VII

### A CONTINUOUS BEAM OF MANY SPANS, FREE AT THE TWO OUTER SUPPORTS, LOADED WITH TWO POINT LOADS AT THE THIRD POINTS

After what has been said in the previous chapters no further explanation of these curves is deemed necessary. Calculation of the bending moments on which they are based is made in Vol. I, Appendix I, 16 and 17, and as before the moments in the end span has been taken to be the same as for the end of three spans, not only for simplicity but also because, owing to the variation of moments of inertia between the end spans and the interior spans, this is more accurate than if any higher number of spans were adopted for this purpose.

Remember when designing that the reaction on the supports one span from the ends may be 20 per cent. greater than $\frac{W}{2}$ for the live load, 10 per cent. for the dead load, $16\frac{2}{3}$ per cent. when live load equals twice dead load, and so on.

On interior supports the reaction may be 20 per cent. higher than $\frac{W}{2}$ for live load but no increase in the case of dead load. This is dealt with in Vol. I.

Remember also when calculating the moments from these diagrams that $W$ is not the value of the point load but is the total load on any one span, and is therefore equal to twice the value of each point load.

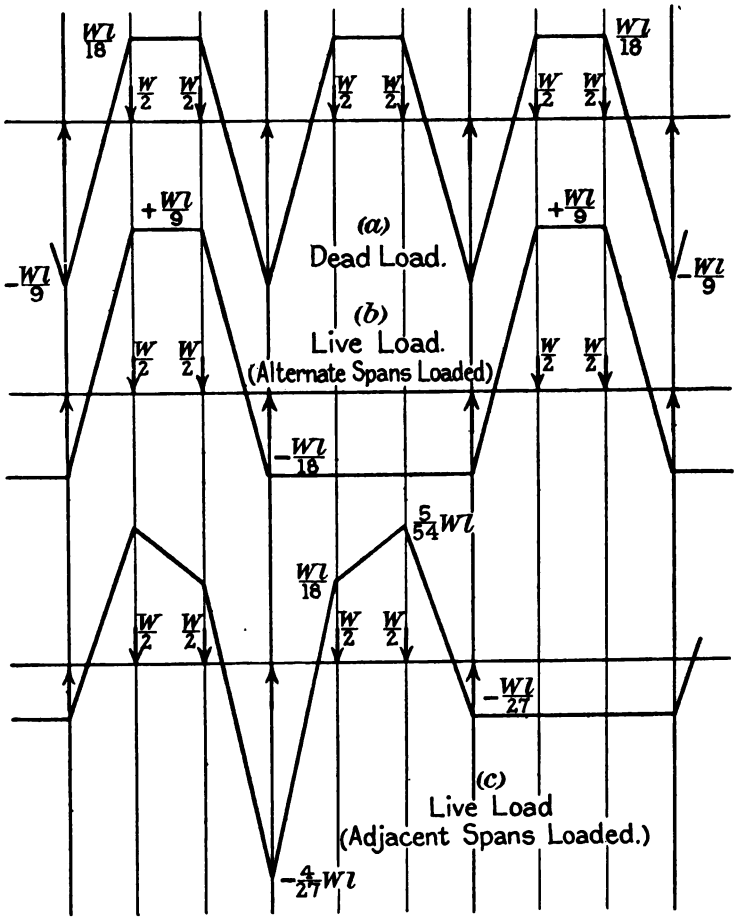



Fig. 38. Curve of Moments. Interior Spans. Two Point Loads at Third Points.

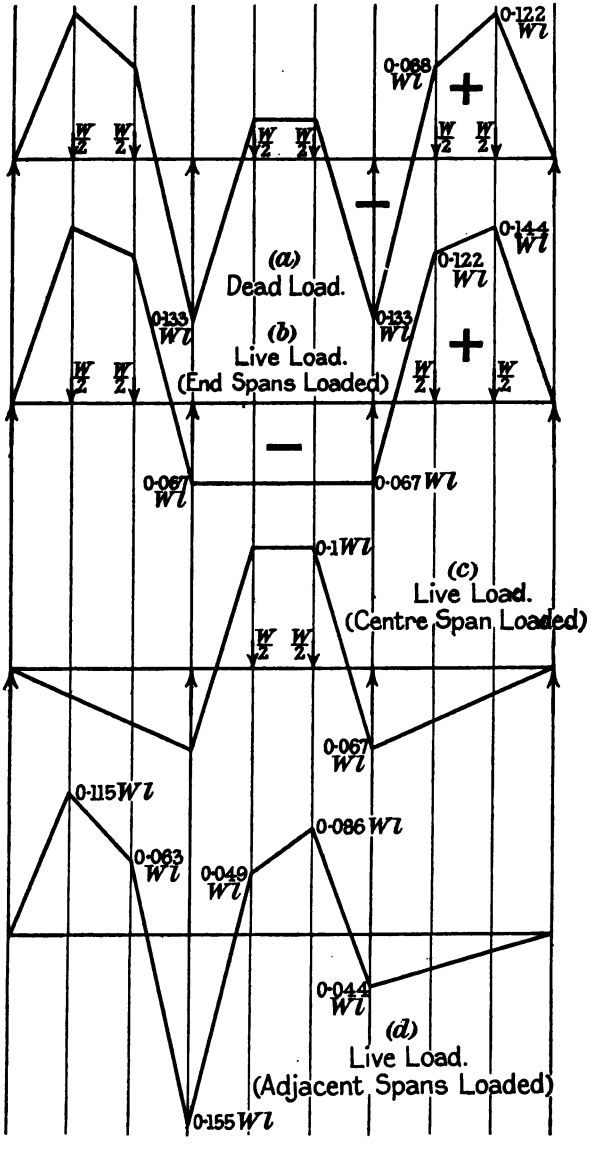



Fig. 39. Curves of Moments. Three Spans [for all End Spans]. Two Point Loads at Third Points.

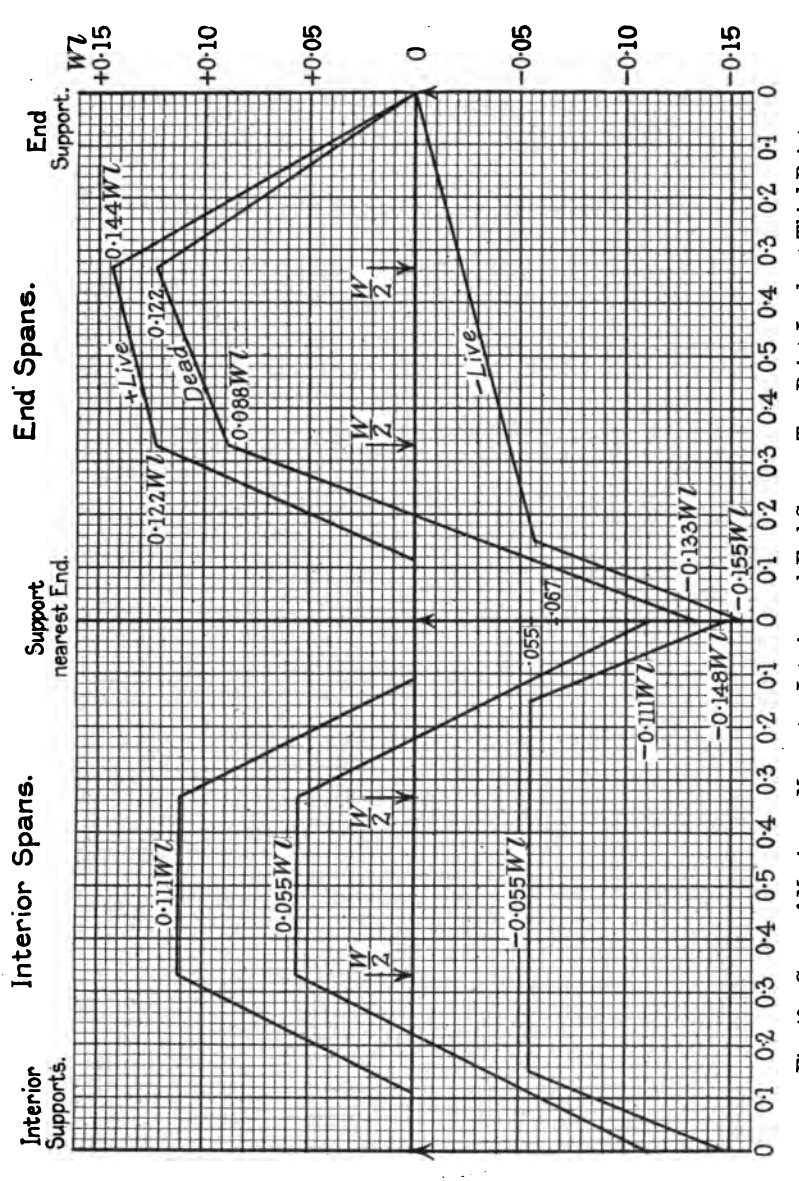



Fig. 40. Curve of Maximum Moments. Interior and End Spans. Two Point Loads at Third Points.

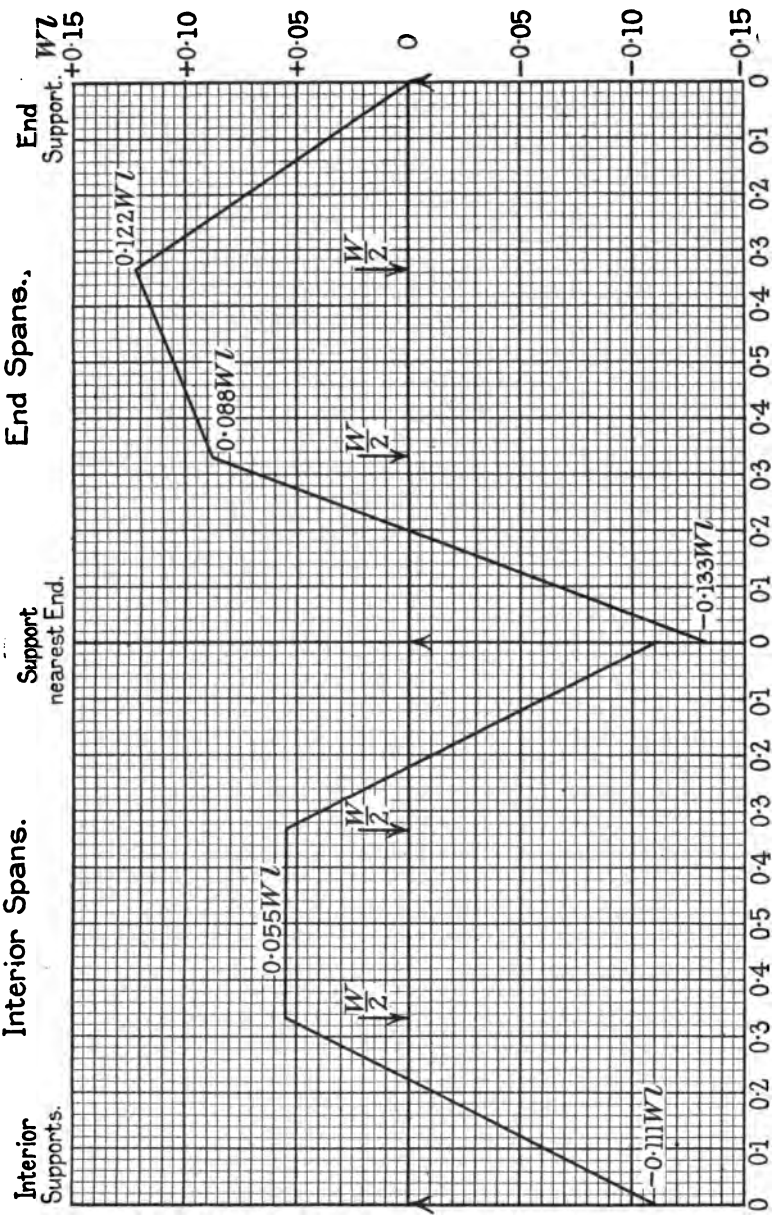



Fig. 41. Curve of Maximum Moments. Interior and End Spans. Two Concentrated Loads at Third Points. Whole Load Dead [Live Load negligible].

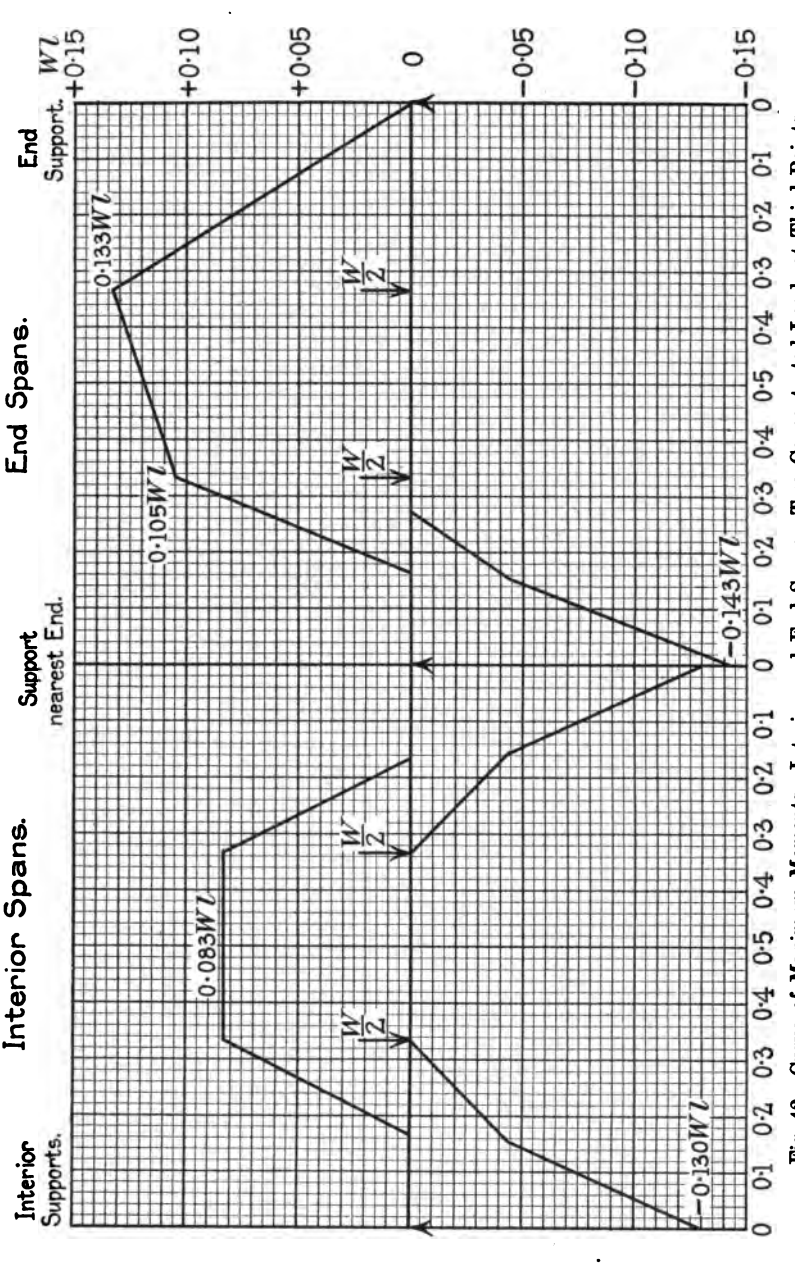



Fig. 42. Curve of Maximum Moments. Interior and End Spans Two Concentrated Loads at Third Points. Live Load = Dead Load.

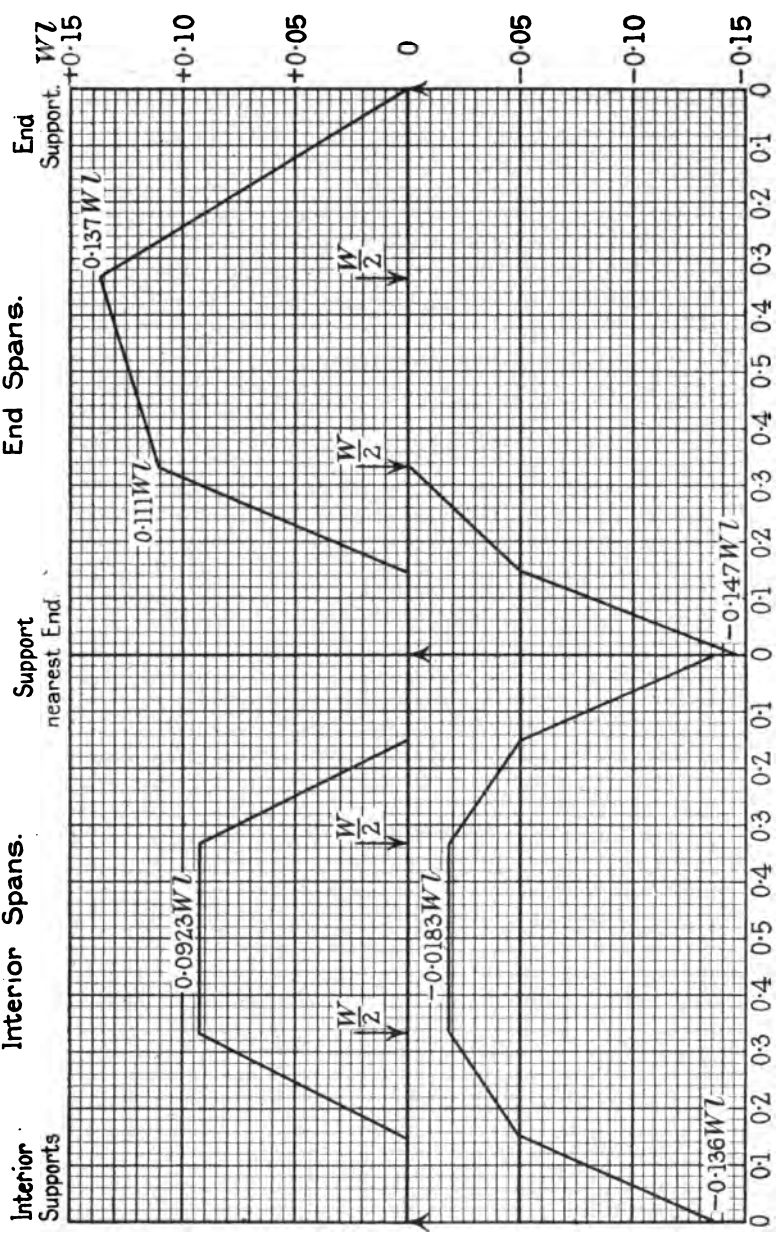



Fig. 43. Curve of Maximum Moments Interior and End Spans. Two Concentrated Loads at Third Points. Live Load = 2 Dead Load.

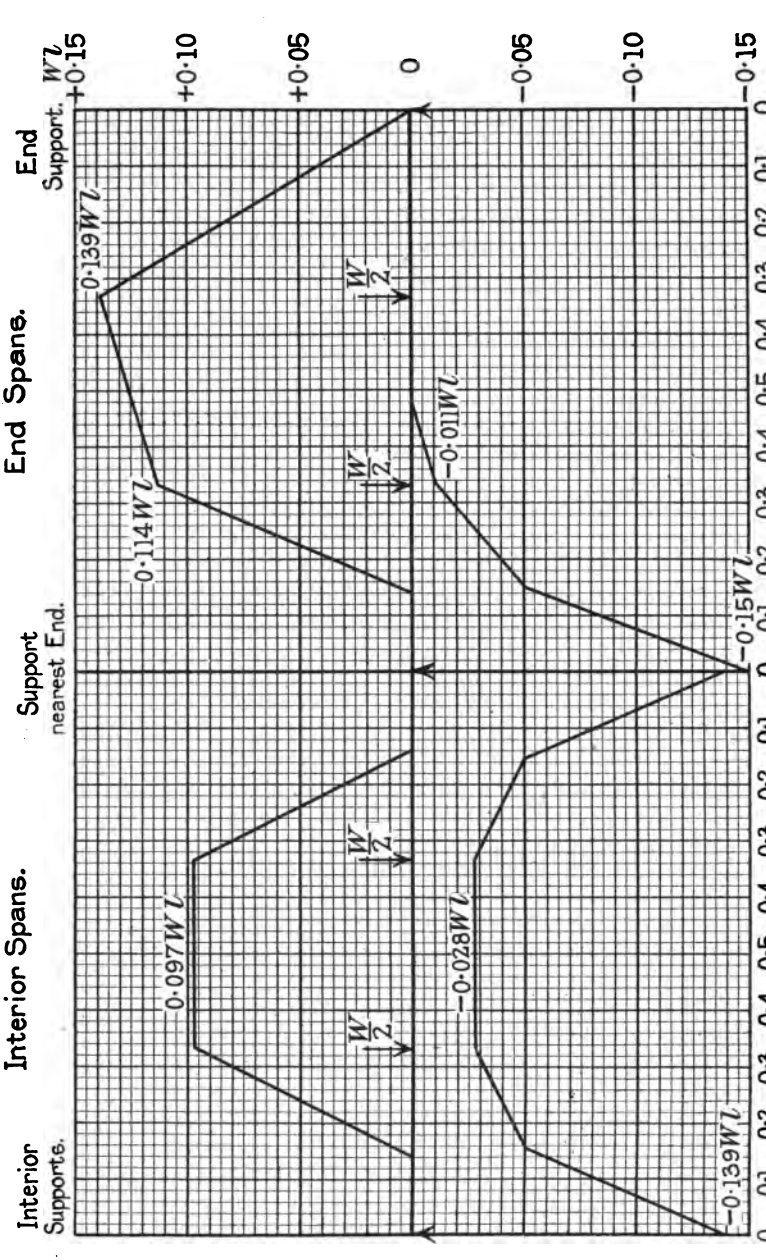



Fig. 44. Curve of Maximum Moments. Interior and End Spans. Two Concentrated Loads at Third Points. Live Load = ⅔ Dead Load.

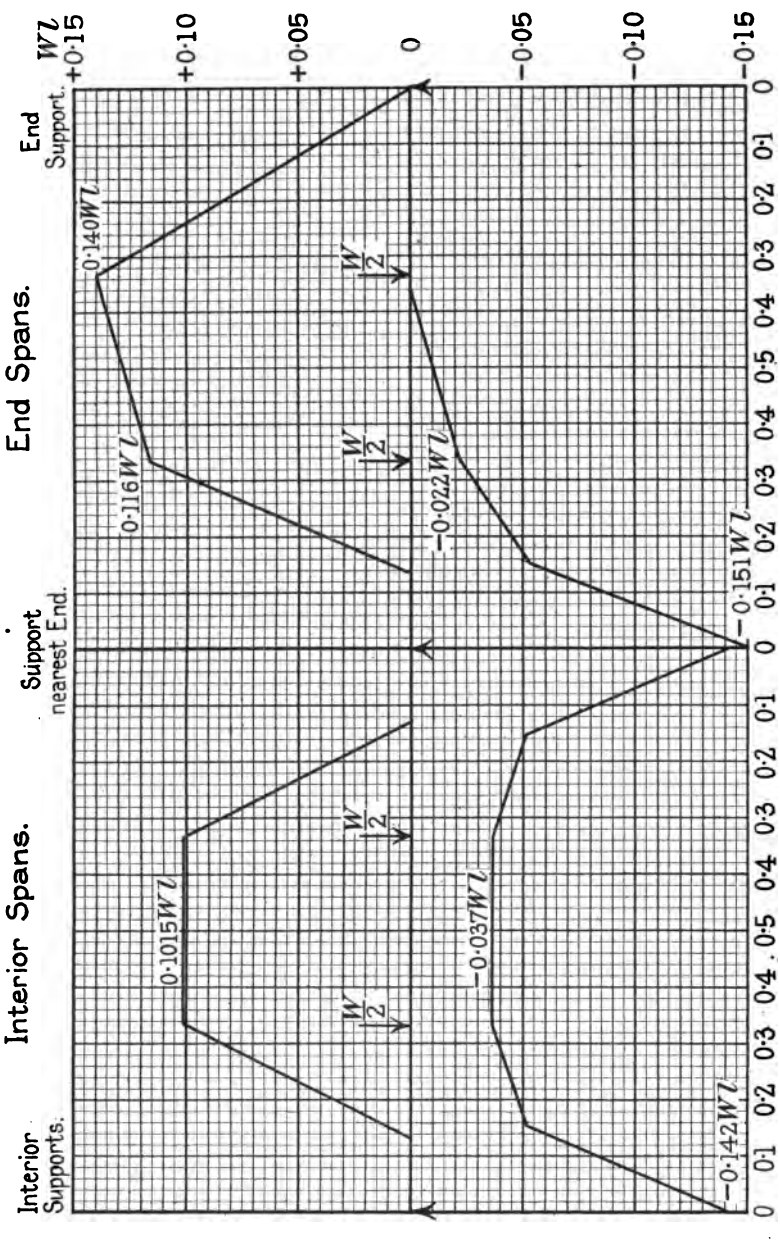



Fig. 45. Curve of Maximum Moments. Interior and End Spans. Two Concentrated Loads at Third Points. Live Load = 5 Dead Load.

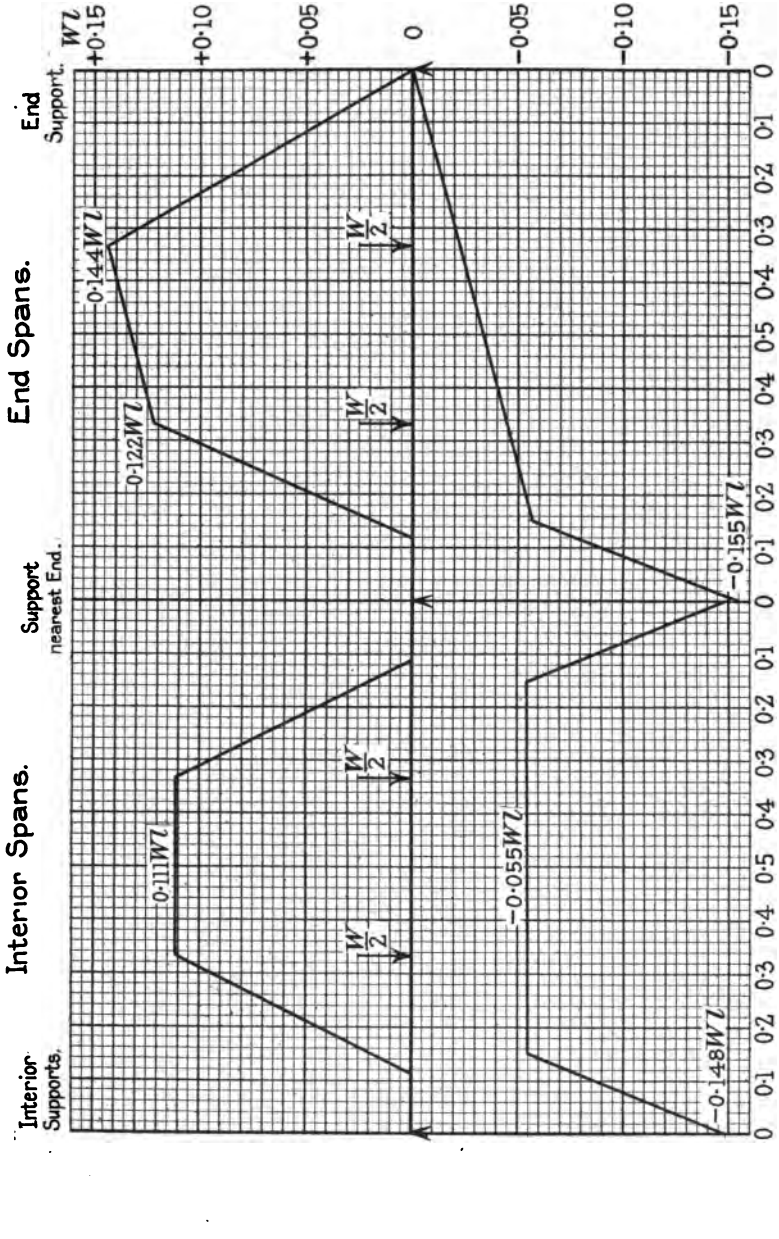



Fig. 46. Curve of Maximum Moments. Interior and End Spans. Two Concentrated Loads at Third Points. Whole Load Live [Dead Load negligible].

# CHAPTER VIII

## OTHER ARRANGEMENTS OF LOADING

We have dealt completely now with the following distributions of load: uniform, point load at mid-span, and two point loads at the third points. These are by far the most common arrangements in practice but others are met with and we must explain how to deal with these.

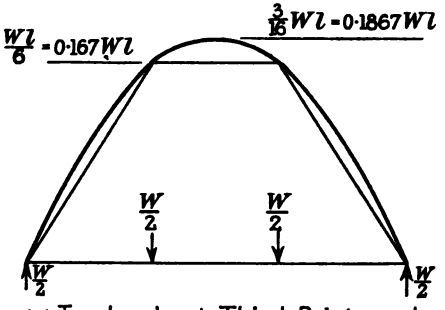
**Equivalent Distributed Load Method.** One method which has been found useful and convenient is to consider an equivalent uniformly distributed load so that the bending moment curve of this equivalent load never lies inside that of the actual loading.

In Fig. 47 is shown exactly what is meant. On the top diagram (*a*) is drawn the bending moment curve for two point loads at the third points and in addition a parabola representing the bending moment curve of the equivalent uniform load of $\frac{8}{9}W$, where $W$ is the total load on the two point loads. It will be seen that at the two point loads the bending moments agree, whereas between them and between them and the supports the moment curve on the uniform load gives slightly greater moments but not so much so as to be uneconomical in design.

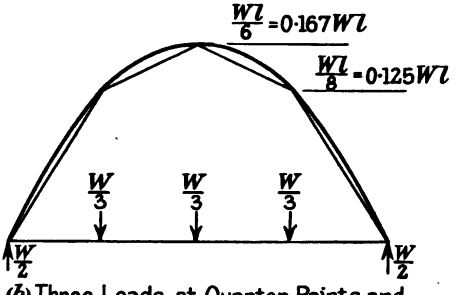
Diagrams (*b*) and (*c*) show the same thing for three point loads at the quarter points and four point loads at the fifth points respectively. The equivalent uniform loads are $\frac{4}{3}W$ and $\frac{5}{4}W$ respectively.

From these results the following general rule may be noticed. The uniform load equivalent to any number of equal point loads spaced at points which divide up the span equally may be derived by multiplying the actual load by the number of divisions into which the span is divided and dividing by the number of point loads.

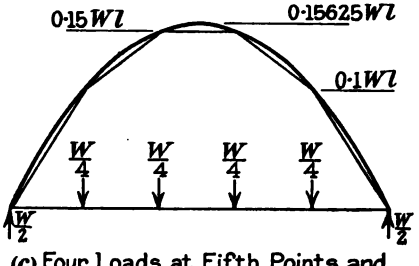
Now the three diagrams on Fig. 47 are all drawn for a beam with ends free and are what we call free moment curves, and it clearly does not follow that because the theorem applies for free moment curves that it applies equally well with con-




(*a*) Two Loads at Third Points and Equivalent Uniform Load of $\frac{3}{2}$W.




(*b*) Three Loads at Quarter Points and Equivalent Uniform Load of $\frac{4}{3}$W.




(*c*) Four Loads at Fifth Points and Equivalent Uniform Load of $\frac{5}{4}$W.

Fig. 47. Moment Curves of Point Loads and of Equivalent Dead Load.

tinuous beams. As an example to test this refer to Fig. 48, in which the bending moment curves for dead load, both for interior spans and end spans are drawn first of all for two point loads at the third points and then for the equivalent uniformly distributed load of $\frac{8}{9}W$. It will be seen that the uniformly distributed load curve agrees sufficiently well with the other for all practical purposes. It will be noticed, however, that the point load curve does in some places give greater results than the uniformly distributed load curve though this was not the case with the free bending moment curves and therefore, while this method is sometimes useful as a rough approximation, there are other methods which are better, and are explained later.

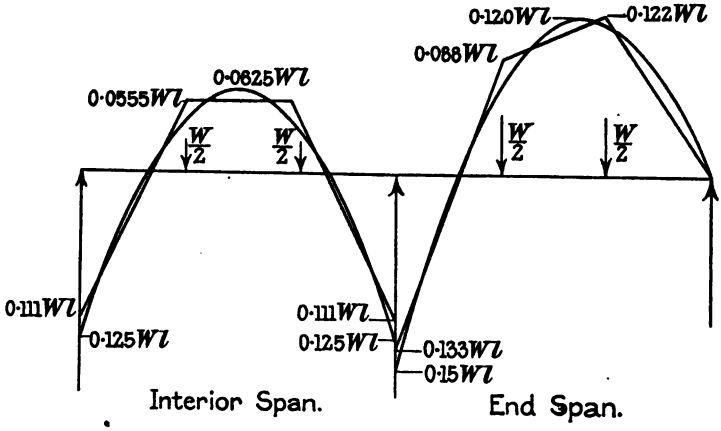



Fig. 48. Curve of Maximum Moments. Interior and End Spans. Two Concentrated Loads at Third Points. Whole Load Dead [from Fig. 41] and of Equivalent Uniform Load of $\frac{3W}{2}$ [from Fig. 23].

**Combination of Curves Method.** It is sometimes found that where the loading differs from one of the three which we have worked out so fully it may be expressed as a combination of them. An example of this is given in Fig. 49. At the top of this, (*a*) shows diagrammatically three bogie wagons, each one occupying one span arranged in the condition for the worst moment at the support between the two adjacent wagons. Now the moments in this case may be obtained by considering

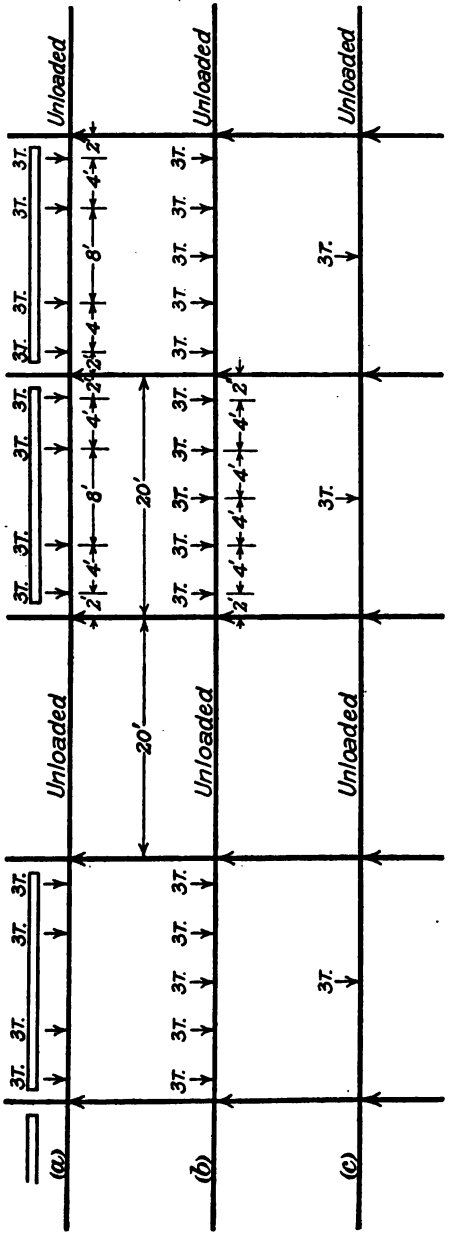



Fig. 49. Wagons on two bays, next bays unloaded, then two more loaded. Arrangement of loads (*a*) may be considered as composed of difference between (*b*) and (*c*) and Moments calculated accordingly.

the arrangement at (*a*) as equivalent to that at (*b*) underneath minus that of (*c*). The advantage of so considering it is that we can then use our curves or formulae direct. Thus the arrangement in (*b*) is clearly the same as for a uniformly distributed load of 15 tons on the span, and that in (*c*) the same as for a point load at mid-span of three tons. The negative moment at the support is therefore

$$
\begin{aligned}
&-\frac{Wl}{9}+\frac{Wl}{6}\\
&=-\frac{15\times 20}{9}+\frac{3\times 20}{6}\\
&=-33{\cdot}3+10 \text{ ft. tons}\\
&=-23{\cdot}3 \text{ ft. tons.}
\end{aligned}
$$

In the above the left-hand expressions will be recognised as the bending moment at the support of a uniformly distributed load with adjacent bays loaded, and those on the right as those of a concentrated load under similar conditions.

It will be necessary to consider the case of one wagon on alternate bays as in Fig. 50 and the moments here may be treated exactly in the same way by considering (*a*) as the equivalent of the arrangement (*b*) minus the arrangement (*c*) and treating the second as the uniform load and the third as the concentrated load at mid-span. Then, working exactly as before, we may write the positive moment at mid-span under the loaded panel equals

$$
\begin{aligned}
&\frac{Wl}{12}-\frac{3Wl}{16}\\
&=\frac{25\times 20}{12}-\frac{3\times 3\times 20}{16}\\
&=25-11{\cdot}1\\
&=13{\cdot}9 \text{ ft. tons.}
\end{aligned}
$$

In the same way the negative moment which may exist in the unloaded span at mid-span is given by

$$
\begin{aligned}
&-\frac{Wl}{24}+\frac{Wl}{16}\\
&=-\frac{15\times 20}{24}+\frac{3\times 20}{16}\\
&=-12{\cdot}5+3{\cdot}75\\
&=-8{\cdot}75 \text{ ft. tons.}
\end{aligned}
$$

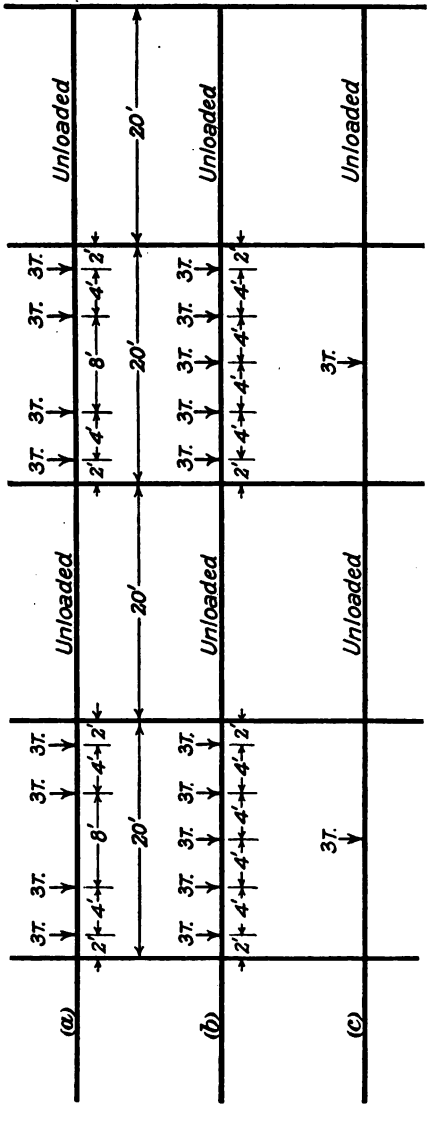



Fig. 50. Wagon on Alternate Bays. Arrangement of Loads (*a*) may be considered as composed of Difference between Arrangements (*b*) and (*c*) and Moments calculated accordingly.

From these results the curve of maximum bending moments due to live load can be calculated for the interior spans exactly as was explained in the earlier chapters.

This method is a very powerful instrument in the hands of a designer with some ingenuity who thoroughly grasps the subject.

Another method of treating cases of loading not so far dealt with is by a graphical construction based on **Clapeyron's Theorem of Three Moments** which may be stated as follows (see Fig. 51):

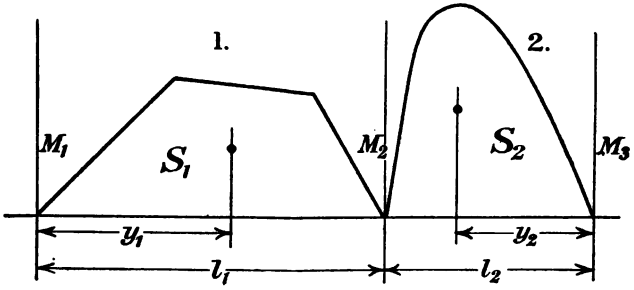



Fig. 51. Illustrating Theorem of Three Moments.

Let $M_1$, $M_2$, $M_3$ be the moments at three consecutive supports of any continuous beam;

$S_1$ the area of the free bending moment curve on span 1;

$S_2$ the area of the free bending moment curve on span 2;

$y_1$ the distance from left support to the centre of gravity of the area $S_1$;

$y_2$ the distance from the right support to the centre of gravity of the area $S_2$.

Then

$$M_1 l_1 + 2M_2 (l_1 + l_2) + M_3 l_2 = 6 \left\{ \frac{S_1 y_1}{l_1} + \frac{S_2 y_2}{l_2} \right\}$$

In the case of actual beams, the formula may be applied successively to spans 1 and 2, 2 and 3, 3 and 4, and so on, and then furnishes enough equations to solve for all moments.

The importance and usefulness of the theorem to a student of continuous beams can hardly be over-estimated. All the foregoing results in which the stiffness of columns was neglected could be derived very simply by its use, and the only reason

we have not done so, is because we wanted the formulae in which the columns are taken into account, for which it is not applicable, and the simple cases are then derived from our formulae by putting $KC = 0$.

We will now use this theorem for deriving very simple rules for determining the moments in continuous beams with unusual load arrangements, confining ourselves to cases in which on any one loaded span the load is symmetrically disposed about the centre line $\left[\text{which means } y = \frac{l}{2}\right]$, and in which the spans are equal [$l_1 = l_2$, etc.], and we will consider interior bays so far removed from the ends, that their influence is not felt.

We have shown that we get the worst conditions for all parts of a beam by considering the following cases:

(*a*) All bays loaded [dead load curve].

(*b*) Alternate bays loaded [live load curves for maximum positive and negative moments near mid-span].

(*c*) Two adjacent bays loaded [live load curve for maximum negative moment at support].

We will now derive simple rules for determining moments for (*a*), (*b*) and (*c*) with any symmetrical loading.

*Case* (*a*). ALL BAYS LOADED.

*Rule. Draw the free bending moment curve. Draw a horizontal line across it, so that the area above the line equals the area below it. Then this will be the bending moment curve.*

Referring to Fig. 52 (*a*), a symmetrical moment curve is shown drawn as a free moment curve on the base $XX$. The particular example taken is one where the load is nothing near mid-span, and piles up near the supports. The line $YY$ is drawn parallel to $XX$ so that the portion $A$ above it is equal in area to the sum of the two portions $B$.

Then $YY$ is the base of the moment curve when continuous and all panels loaded, $A$ is the positive moment and $B$ the negative moment.

*Proof.* Applying the theorem of three moments to bays 1 and 2 in Fig. 52 (*b*) and remembering that owing to symmetry

$$M_1 = M_2, \text{ etc. [call them } M],$$

$$S_1 = S_2, \text{ etc. [call them } S].$$

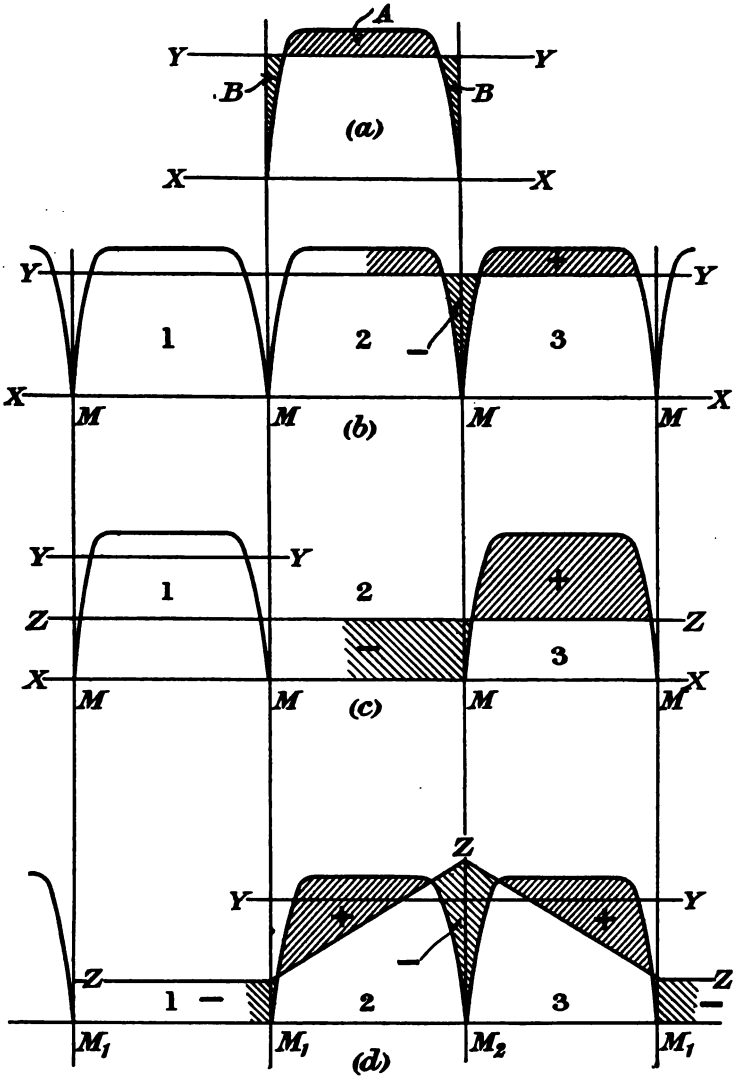



Fig. 52. Bending Moment Curves for special cases by Theorem of Three Moments.

Then $$Ml + 4Ml + Ml = 6S$$

$$\therefore M = \frac{S}{l}$$

$\therefore M$ is the average ordinate of the area $S$.

*Check.* As a check, apply this to the case of uniform loading.

Here, the area of the parabola of height $\frac{Wl}{8}$ and base $l$ is

$$S = \frac{2}{3} \cdot \frac{Wl}{8} \cdot l$$

$$= \frac{Wl^2}{12}$$

$$\therefore M = \frac{S}{l} = \frac{Wl}{12}$$

A result which we recognise.

*Case (b).* ALTERNATE BAYS LOADED.

*Rule. Draw the free bending moment curve. Draw a horizontal line across it, so that the area above the line equals the area below it.*

*Then a line half-way between the original base and the new base is the base line of the continuous beam.*

In Fig. 52 (*c*) the free moment curve is shown drawn on alternate panels, and the average ordinate line $YY$ as in Fig. 52 (*a*).

Then the line $ZZ$, half-way between $YY$ and $XX$ is the base of the moment diagram of the continuous beam.

*Proof.* Applying theorem of three moments to spans 1 and 2, remembering that from symmetry

$$M_1 = M_2, \text{ etc. [call them } M]$$

$$S_2 = 0.$$

Then $$Ml + 4Ml + Ml = \frac{6S}{2} + 0$$

$$= 3S$$

whence $$M = \frac{S}{2l}$$

Hence $M$ is half the mean ordinate, $\frac{S}{l}$.

*Check.* Applying this to the case of uniform loading, we showed that the mean ordinate was $\frac{Wl}{12}$.

Hence $M = \frac{Wl}{24}$, which we recognise.

*Case (c).* TWO ADJACENT BAYS LOADED.

*Rule. Draw the free moment curves on XX and the line YY as before. Then the moment $M_2$ will be $\frac{4}{3}$ times the mean ordinate, and $M_1$ will be $\frac{1}{3}$ times the mean ordinate.*

*Proof.* In Fig. 52 (*d*) the free moment curve is drawn on two consecutive bays, and the line *YY*, the mean ordinate, as in Fig. 52 (*a*).

The equality of the moments $M_1$ follows from symmetry.

Applying the theorem of three moments to bays 1 and 2

$$M_1 l + 2M_1 (2l) + M_2 l = 3S$$

whence
$$5M_1 + M_2 = 3\frac{S}{l} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Applying it to bays 2 and 3,

$$M_1 l + 2M_2 (2l) + M_1 l = 6\frac{S}{l}$$

whence
$$4M_1 + 2M_1 = 6\frac{S}{l} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Multiply (1) by 4,
$$4M_2 + 20M_1 = 12\frac{S}{l} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Subtract (2) from (3),
$$18M_1 = 6\frac{S}{l}$$

$$M_1 = \frac{1}{3} \cdot \frac{S}{l} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

in accordance with rule.

Substitute (4) in (1),

$$M_2 = 3\frac{S}{l} - \frac{5}{3} \cdot \frac{S}{l}$$

$$= \frac{4}{3} \cdot \frac{S}{l} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

in accordance with rule.

*Check.* Applying this to uniform load, our mean ordinate $\frac{S}{l}$ is $\frac{Wl}{12}$ as before.

Therefore $$M_1 = \frac{1}{3} \cdot \frac{Wl}{12}$$
$$= \frac{Wl}{36}$$

and $$M_2 = \frac{4}{3} \cdot \frac{Wl}{12}$$
$$= \frac{Wl}{9}$$

two results which we recognise.

These rules are extremely easy and convenient to use in practice. As a rule, the estimation of the mean ordinate may be done with sufficient accuracy by eye, but the result can be checked if desired by measuring the areas $A$ and $B$ in Fig 52 ($a$) with a planimeter and seeing that $A = 2B$.

*End panels* may be treated with sufficient accuracy as before, that is by considering the moment as the support nearest the wall the same as for the interior supports under cases ($a$), ($b$) and ($c$), and joining this point to the intersection of the original axis $XX$ with the wall support, and erecting the suitable free moment diagram on this line as base.

# CHAPTER IX

## ROLLING LOADS

It is well known that with ordinary free beams, rolling loads produce different moment curves of maximum moments from those produced by equal but fixed loads.

Thus in Fig. 53 the maximum moment occurs with the load at mid-span, and the moment curve is then the straight line diagram (*b*).

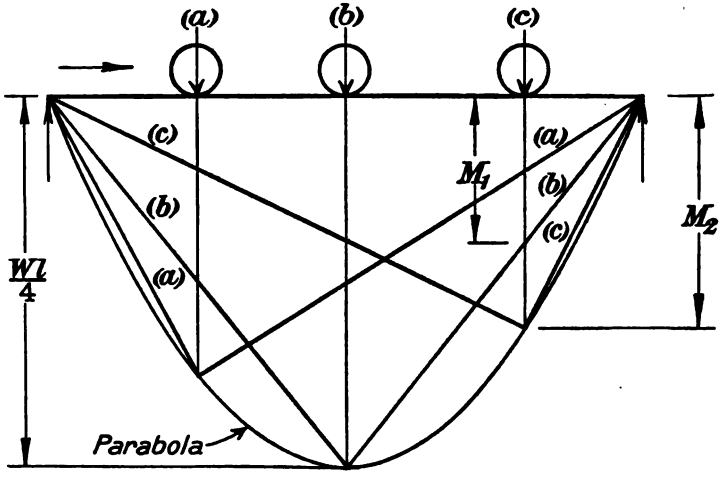



Fig. 53. Moment due to Single Rolling Load.

Now if the load were a fixed one, this moment curve would not be exceeded, but with a rolling load, the load may be either at (*a*) or at (*c*) at different times, the moment curves then being (*a*) or (*c*) respectively, which, it will be noticed, lie outside the moment curve (*b*).

It is well known that the apexes of all such moment diagrams lie on a parabola as drawn.

In other words, while two beams designed to carry a certain point load, fixed at mid-span in the one, and free to roll anywhere

in the other, would be designed for the same moment at mid-span, the second would need to be designed for greater moments at all other points, the curves being the curve (*b*) and the parabola on Fig. 53, respectively.

All this is well understood.

With several moving loads at definite distances apart—such as the wheels of a train—curves of maximum moment can be constructed corresponding to the parabola in Fig. 53, but consist now of several superimposed parabolas with cusps at points determined by the position at which additional wheels roll onto the span. All this may be studied in books on civil engineering, and much ingenuity has been expended in devising methods of "influence lines," etc., which for the free beams for which they were intended were very useful. Unfortunately all this technique is of little use with continuous beams, because a more important consideration enters.

With free beams, the value and position of the second load in a train does not affect the moment on the span on which the first load has rolled until it, in turn, also rolls onto the span. But with continuous beams it does affect these moments very greatly.

Thus, consider a viaduct as in Fig. 54. A point load on span 1, with no load on either adjacent span, might produce a positive moment at mid-span of

$$M = \cdot 1875Wl \text{ (see Fig. 31)}$$

but if there is an equal load on the centre of each adjacent span, the moment is only

$$M = \cdot 125Wl \text{ (see Fig. 31)}$$

a reduction of $33\frac{1}{3}$ per cent.

Similarly if there is a group of loads on one span, and none on the adjacent spans, the $M_c$ may approximate

$$\frac{2}{3}\frac{Wl}{8} \text{ [see Fig. 20]}$$

whereas with loads on adjacent panels

$$M_c = \frac{1}{3}\frac{Wl}{8}$$

a reduction of 50 per cent. [See Fig. 20.]

Thus, in Fig. 54 the moment due to load 1 on span 1 is considerably less than if load 2 were not present on span 2, and the same reduction applies to the moment due to load 2 on span 2, owing to presence of load 1 on span 1.

The co-existence of these several loads at definite distances often prevents alternate bays being loaded and unloaded, the worst live load conditions in the previous chapters, and hence moments often approximate more to dead load than live load curves, with a greater reduction of centre moments, due to continuity, and lesser negative moments at mid-span for the same reason, but not much change to negative moments at support (which did not vary so greatly between live and dead load conditions).

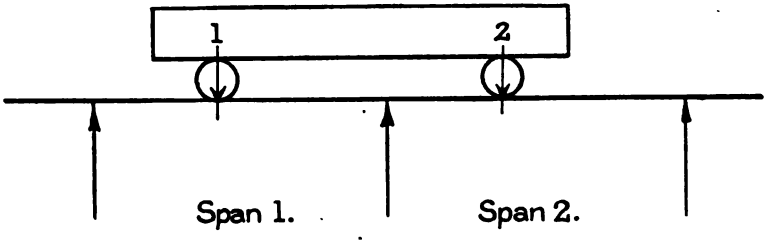



Fig. 54. Rolling Load on Two Spans.

We may distinguish the following cases.

(1) When the spans are long in comparison with the length of trucks.

Then we may have any span loaded and adjoining spans unloaded, and the ordinary formulae and methods apply without modification.

(2) When the spans are approximately the length of one truck.

Consider Fig. 55 (*a*) and (*b*), showing a truck say 20′ long on a 20′ span with a wheelbase of say 16′.

It is clear that one truck may rest on a span, without load on adjoining spans, as at (*a*) for which the ordinary formulae apply.

But the disposition of loads, as at (*a*), may not be the condition which produces greatest free moments. For example, at Fig. 55 (*b*) is a condition in which the free bending moment is

clearly much greater, since we still have two axles on one span, but disposed much nearer to mid-span.

Thus, considering the spans 20′, length of truck 20′, wheel-base 16′, load on each axle $10^T$, the free bending moment in arrangement (*a*) would be

$$10^T \times 2' = 20 \text{ ft. tons}$$

while in (*b*) it would be

$$10^T \times 8' = 80 \text{ ft. tons.}$$

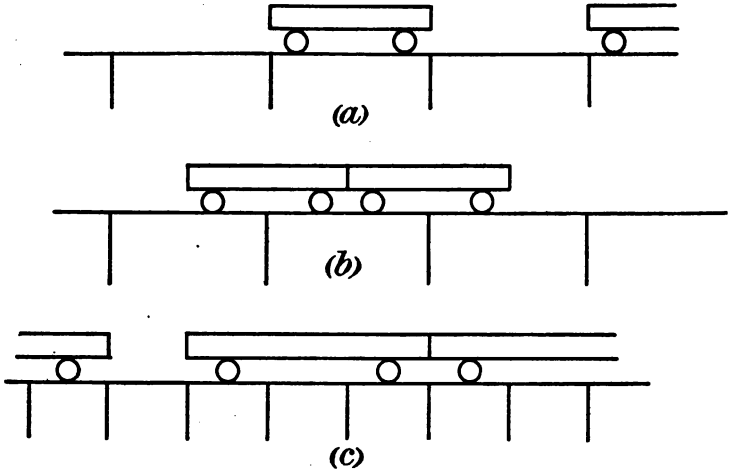



Fig. 55. Arrangements of Trucks on Spans.

Arrangement (*b*) is clearly the one to design for; and this can *not* occur without load on adjoining panels, and need therefore *not* be designed for this condition, which means a smaller bending moment at the centre, since good continuity is assured.

(3) When the spans are much shorter than a truck.

This generally occurs where a line is supported by cross girders.

In this case, we may have the most damaging conditions of alternate bays loaded, and also of two adjacent bays loaded, and all our previous formulae apply for the maximum moment at mid-span or support, but the intermediate portions must be designed more liberally for positive and negative moments than if fixed point loads at mid-span were the only loads, because of the consideration of moving loads illustrated by Fig. 53, in which the worst moment at (*c*) is $M_2$ instead of $M_1$.

# CHAPTER X

## UNEQUAL SPANS

A finite number of Unequal Spans may always be investigated by the method of the theorem of three moments. Unfortunately, the amount of calculation required with four spans and over is most excessive since with many spans we have to consider not only the three conditions

(*a*) all spans loaded,

(*b*) alternate spans loaded,

(*c*) two adjacent spans loaded,

but there are clearly, with four unequal spans, two cases of (*b*) and three of (*c*) to investigate, and each is cumbersome except in very simple cases, and with increasing number of spans, the number of cases requiring consideration rises very rapidly.

The author has, however, been able to reduce the whole calculation into terms of very simple Factors, for which tabulated and plotted values are given. These are believed to be original and so convenient, as to make a reasonably accurate solution of many unequal spans a practical matter for the first time.

The convenience and importance of these factors is that they apply alike to any number of spans, and different arrangements of loading, and even apply to cases of many spans all unequal with quite sufficient accuracy.

**Continuous Beam of Two Spans, ends freely supported, not fixed. Uniform load.** Clearly the moments are all determined if we know $M_2$, the moment at the interior support.

There are three cases to consider:

(*a*) both spans loaded,

(*b*) long span only loaded,

(*c*) short span only loaded.

*Rule.* *The moment at the interior support* $M_2$ *is*

$$-\frac{Wl}{8} \cdot F_1 \text{ in case } (a)$$

$$-\frac{Wl}{8} \cdot F_2 \text{ in case } (b)$$

$$-\frac{Wl}{8} \cdot F_3 \text{ in case } (c)$$

*where* $W$ *is the load on the long span,*
$l$ *is the long span,*

*and*

$$F_1 = 1 - \frac{l_1}{l_2} + \left(\frac{l_1}{l_2}\right)^2$$

$$F_2 = \frac{l_2}{l_2 + l_1}$$

$$F_3 = \left(\frac{l_1}{l_2}\right)^2 \cdot \frac{l_1}{l_2 + l_1}$$

*where* $l_1$ = *short span and* $l_2$ = *long span.*

*It is not necessary to calculate* $F_1$, $F_2$ *or* $F_3$, *since they can be taken at once from the following table, or from Fig.* 56.

TABLE I.

Values of Factors $F_1$, $F_2$, $F_3$.

| Ratio of spans | $F_1$ | $F_2$ | $F_3$ |
|---|---|---|---|
| $\frac{l_1}{l_2}$ | $1-\frac{l_1}{l_2}+\left(\frac{l_1}{l_2}\right)^2$ | $\frac{l_2}{l_2+l_1}$ | $\left(\frac{l_1}{l_2}\right)^2 \cdot \frac{l_1}{l_2+l_1}$ |
| 1 | 1·0 | ·5 | ·5 |
| ·9 | ·91 | ·525 | ·384 |
| ·8 | ·84 | ·555 | ·285 |
| ·7 | ·79 | ·588 | ·207 |
| ·6 | ·76 | ·625 | ·135 |
| ·5 | ·75 | ·666 | ·083 |
| ·4 | ·76 | ·714 | ·046 |
| ·3 | ·79 | ·769 | ·021 |
| ·2 | ·84 | ·833 | ·006 |
| ·1 | ·91 | ·91 | ·001 |
| 0 | 1·0 | 1·0 | 0 |

*Proof.* The following assumptions are made:

1. *The moment of inertia of the beam is the same in both spans.*

This is approximately true in practice, since the moments in the short span are chiefly determined by the long one, so

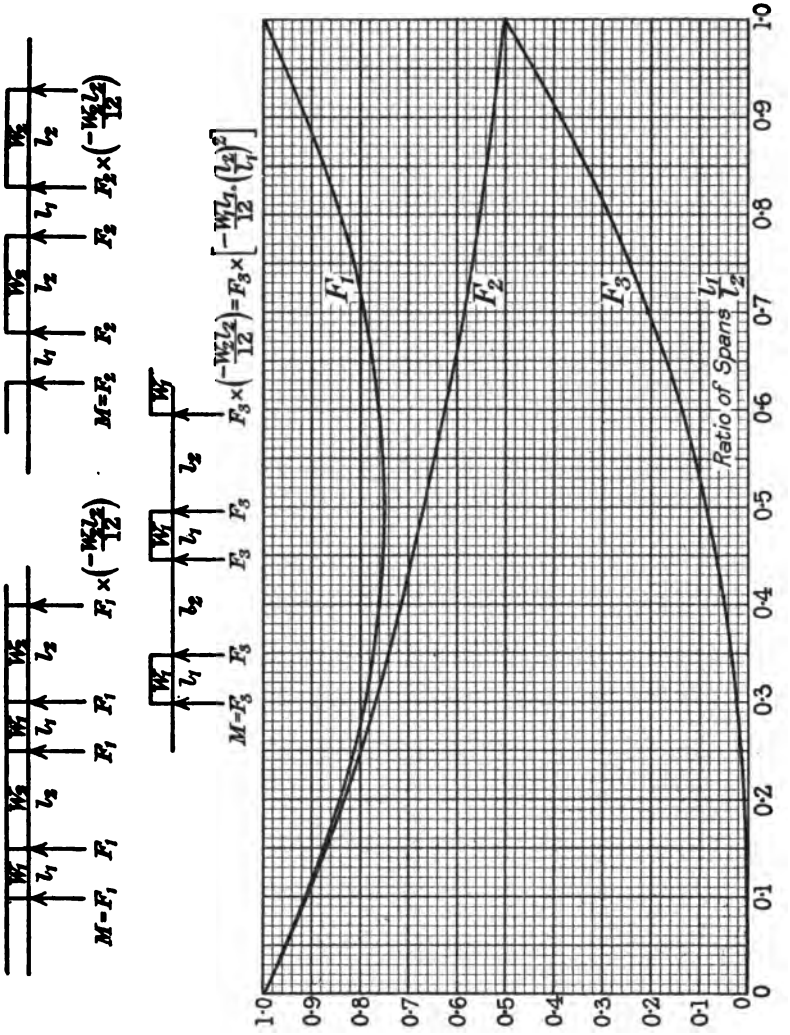



Fig. 56. Factors $F_1$, $F_2$, $F_3$ for Moments at Supports with Unequal Spans.

that the average moment [without regard to sign] in the two spans is nearly the same.

2. *The load is the same per ft. run in the two spans* [though they need not be loaded simultaneously] so that

$$\frac{W_1}{W_2} = \frac{l_1}{l_2}$$

This condition would be met whenever the beam under consideration supports a slab of constant span, and would generally apply.

Adopting the notation of *Reinforced Concrete Design*, Vol. I, Appendix I, we have

$$B_1 = \frac{I_1}{l_1} = \frac{\text{moment of inertia}}{\text{span}} \text{ of beam 1}$$

$$B_2 = \frac{I_2}{l_2} = \frac{\text{moment of inertia}}{\text{span}} \text{ of beam 2}$$

Then, since from Assumption 1, $I_1 = I_2$, we have

$$\frac{B_1}{B_2} = \frac{l_2}{l_1} \text{ or } B_1 = B_2 \cdot \frac{l_2}{l_1}$$

Now, from Appendix I, we get two equations for $M_1$ and $M_2$ (using notation of Fig. 57 (*a*)).

Applying these successively to left-hand and right-hand spans of Fig. 57 (*b*) we get

$$\begin{cases} M_1 = -\frac{W_1 l_1}{12} - 4EB_1\alpha_1 - 2EB_1\alpha_2 = 0 \quad . \quad . \quad (1) \\ M_2 = -\frac{W_1 l_1}{12} + 4EB_1\alpha_2 + 2EB_1\alpha_1 \quad . \quad . \quad . \quad (2) \end{cases}$$

$$\begin{cases} M_2 = -\frac{W_2 l_2}{12} - 4EB_2\alpha_2 - 2EB_2\alpha_3 \quad . \quad . \quad . \quad (3) \\ M_3 = -\frac{W_2 l_2}{12} + 4EB_2\alpha_3 + 2EB_2\alpha_2 = 0 \quad . \quad . \quad (4) \end{cases}$$

From (1) $$4EB_1\alpha_1 = -\frac{W_1 l_1}{12} - 2EB_1\alpha_2$$

Substituting in (2)

$$M_2 = -\frac{W_1 l_1}{12} + 4EB_1\alpha_2 - \frac{W_1 l_1}{24} - EB_1\alpha_2$$

$$= -\frac{W_1 l_1}{8} + 3EB_1\alpha_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Similarly from (3) and (4),

$$M_2 = -\frac{W_2 l_2}{8} - 3EB_2 a_2 \quad . \quad . \quad . \quad . \quad (6)$$

$$\therefore -\frac{W_1 l_1}{8} + 3EB_1 a_2 = -\frac{W_2 l_2}{8} - 3EB_2 a_2$$

But $$B_1 = B_2 \frac{l_2}{l_1}$$

$$\therefore -\frac{W_1 l_1}{8} + 3EB_2 a_2 \left(\frac{l_2}{l_1}\right) = -\frac{W_2 l_2}{8} - 3EB_2 a_2$$

$$3EB_2 a_2 \left(\frac{l_2}{l_1} + 1\right) = \frac{W_1 l_1}{8} - \frac{W_2 l_2}{8}$$

$$3EB_2 a_2 = \frac{W_1 l_1 - W_2 l_2}{8} \cdot \frac{l_1}{l_2 + l_1}$$

whence $$M_2 = -\frac{W_2 l_2}{8} - \frac{W_1 l_1 - W_2 l_2}{8} \cdot \frac{l_1}{l_2 + l_1} \quad . \quad . \quad (7)$$

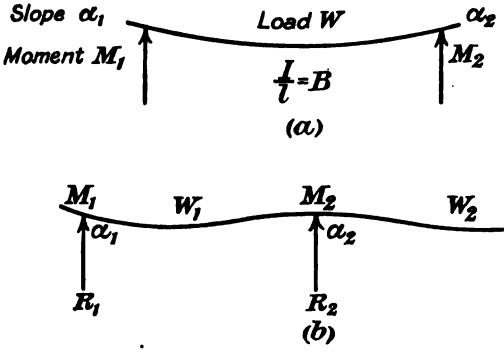



Fig. 57. Continuous Beams. Two Spans. Free at Ends. Notation.

*Case* (*a*). BOTH SPANS LOADED.

$$\frac{W_1}{W_2} = \frac{l_1}{l_2} \quad \therefore \frac{W_1 l_1}{W_2 l_2} = \left(\frac{l_1}{l_2}\right)^2 \quad \therefore W_1 l_1 = W_2 l_2 \cdot \left(\frac{l_1}{l_2}\right)^2$$

Substituting in (7),

$$M_2 = -\frac{W_2 l_2}{8}\left[1 - \frac{l_1}{l_2} + \left(\frac{l_1}{l_2}\right)^2\right]$$

$$= -\frac{W_2 l_2}{8} \cdot F_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Note 1. When $l_1 = l_2$, $F_1 = 1$ and $M_2 = -\frac{W_2 l_2}{8}$, the well-known moment at the central support for two equal spans fully loaded.

Note 2. When $l_1 = 0$, $F_1 = 1$ and $M_2 = -\frac{W_2 l_2}{8}$.

This is also clearly correct, since as the left-hand support approaches more and more nearly to the centre one, the right-hand span is more and more nearly fixed horizontally at the left end, and for $l_1 = 0$ is completely so fixed. $-\frac{Wl}{8}$ is the correct moment at the built-in end for a beam uniformly loaded and fixed one end and free the other.

*Case* (*b*). LONG SPAN ONLY LOADED.

Here $W_1 l_1 = 0$.

Substitute this in (7), we have

$$M_2 = -\frac{W_2 l_2}{8}\left(1 - \frac{l_1}{l_2 + l_1}\right)$$

$$= -\frac{W_2 l_2}{8}\left(\frac{l_2}{l_2 + l_1}\right) \quad \ldots\ldots\ldots \quad (9)$$

$$= -\frac{W_2 l_2}{8} \cdot F_2$$

Note. Putting $l_1 = l_2$ and $l_1 = 0$ we also get well-known values in these simple special cases.

*Case* (*c*). SHORT SPAN ONLY LOADED.

Putting $W_2 l_2 = 0$ in equation (7)

$$M_2 = -\frac{W_1 l_1}{8} \cdot \frac{l_1}{l_2 + l_1}$$

It is convenient to keep all moments in terms of $W_2 l_2$, and therefore we will express

$$M_2 = -\frac{W_2 l_2}{8} \cdot \left(\frac{l_1}{l_2}\right)^2 \cdot \frac{l_1}{l_2 + l_1}$$

$$= -\frac{W_2 l_2}{8} \cdot F_3 \quad \ldots\ldots\ldots \quad (10)$$

This also gives known results in the two special cases.

*Example.* As an example of the use of these factors, and to show how simple is the solution when they are used, consider the following case, when $l_1 = \frac{1}{2}l_2$, $W_1 = \frac{1}{2}W_2$.

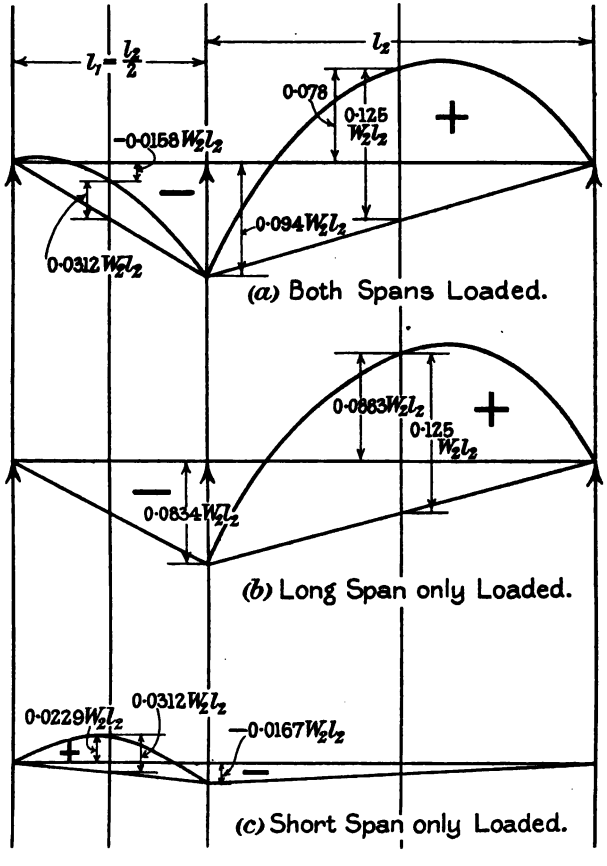



Fig. 58. Maximum Moments. Continuous Beam. Ends Free. Ratio of Lengths 1 : 2.

Then, using either the table or the curve, we have

*Case* (*a*). BOTH SPANS LOADED.

$$M_2 = -\frac{W_2l_2}{8} \cdot F_1$$

$$= -\frac{W_2l_2}{8} \times \cdot 75 = -\cdot 094 W_2l_2$$

Setting this down graphically in Fig. 58 (*a*), and joining this to the intersection of the base line with the two end supports, we derive two sloping base lines on which we construct two parabolas of height

$$\frac{W_2 l_2}{8} = \cdot 125 W_2 l_2 \text{ on the long span}$$

and $$\frac{W_1 l_1}{8} = \frac{W_2 l_2}{32} = \cdot 0312 W_2 l_2 \text{ on the short span.}$$

*Case* (*b*). LONG SPAN ONLY LOADED.

$$M_2 = -\frac{W_2 l_2}{8} \cdot F_2 = -\frac{W_2 l_2}{8} \times \cdot 666$$
$$= -\cdot 0834 W_2 l_2$$

Setting this down graphically, in Fig. 58 (*b*), we derive two sloping base lines as before. The left is the line of negative moments in the short span (except quite near the cusp, where the value is slightly exceeded in (*a*)), and on the right base line we erect a parabola of height $\frac{W_2 l_2}{8} = \cdot 125 W_2 l_2$ as before.

*Case* (*c*). SHORT SPAN ONLY LOADED.

$$M_2 = -\frac{W_2 l_2}{8} \cdot F_3 = -\frac{W_2 l_2}{8} \times \cdot 083 = -\cdot 0167 W_2 l_2$$

Setting this down graphically in Fig. 58 (*c*), the right-hand base line so derived is the maximum negative moment curve for the right-hand span, except near the cusp, where higher values are given from Fig. 58 (*a*). On the right-hand base we erect a parabola of height

$$\frac{W_1 l_1}{8} = \frac{W_2 l_2}{32} = \cdot 0312 W_2 l_2 \text{ as before.}$$

These three curves enable us to design each section of the beam for the worst moments it will be called upon to resist.

**Continuous Beam of Two Spans, ends free, with concentrated load at mid-span.** We will assume, as before, that

(1) The Moment of Inertia is the same through both spans.

(2) The load on the two spans is proportional to the spans.

This would be obtained by the arrangement shown in Fig. 59 with a uniform load per square foot on the floor.

The investigation is exactly similar to the last, except that the basis formulae (from *Reinforced Concrete Design*, Vol. I, Appendix I, p. 269, formulae (7) and (8)) now read (referring to Fig. 57):

$$\begin{cases} M_1 = -\frac{W_1 l_1}{8} - 4EB_1\alpha_1 - 2EB_1\alpha_2 = 0 \\ M_2 = -\frac{W_1 l_1}{8} + 4EB_1\alpha_2 + 2EB_1\alpha_1 \end{cases}$$

$$\begin{cases} M_2 = -\frac{W_2 l_2}{8} - 4EB_2\alpha_2 - 2EB_2\alpha_3 \\ M_3 = -\frac{W_2 l_2}{8} + 4EB_2\alpha_3 + 2EB_2\alpha_2 = 0 \end{cases}$$

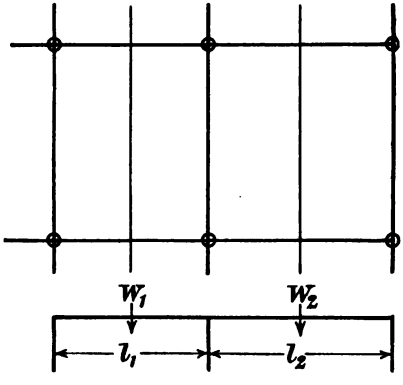



Fig. 59. Arrangement of Beams giving Central Point Loads at Mid-span of Unequal Spans so that $\frac{W_1}{W_2} = \frac{l_1}{l_2}$.

These formulae are clearly identical with the last, except that we now have $\frac{Wl}{8}$ instead of $\frac{Wl}{12}$.

If the same treatment is followed through, it will be found to lead to exactly similar expressions for $M_2$ as before, with the same factors $F_1$, $F_2$ and $F_3$ as before, but with $\frac{3}{16}W_2l_2$ instead of $\frac{1}{8}W_2l_2$.

**Other conditions of loading.** In the same way, if we consider other conditions of loading, such as two point loads at the third points, and make the same assumptions as before,

we start off with exactly similar equations, and derive similar expressions for the support moment, in which the same factors occur, the only difference being the coefficient of $Wl$ in the expressions. In all cases, it will be found that this coefficient is that which applies to the moment at the fixed end of a beam loaded in accordance with the case under consideration, and fixed horizontally at one end and free at the other.

We may therefore write down *generally* as follows:

**General Rule for a Continuous Beam of Two Spans, free at the ends under all conditions of loading.**

*Moment at interior support*

$$M_2 = F_1 M \text{ both spans loaded,}$$
$$= F_2 M \text{ long span only loaded,}$$
$$= F_3 M \text{ short span only loaded.}$$

*$F_1$, $F_2$, $F_3$ are the factors whose values were tabulated and plotted in Table I and Fig.* 56.

*$M$ is the moment at the fixed end of a beam fixed one end, free at the other, and loaded as the case under consideration, and has the following values:*

$\frac{Wl}{8}$ *when the load is uniform,*

$\frac{3}{16}Wl$ „ „ „ „ *concentrated at mid-span,*

$\frac{1}{6}Wl$ „ „ „ „ *concentrated at the two third points.*

**Many Spans.** Any particular case of many spans can be solved accurately by the application of the theorem of three moments, but, as explained, so many cases need to be solved that it is too cumbersome.

What is wanted is a method which can, with reasonable and sufficient accuracy, be used rapidly without a separate mathematical investigation for each case.

The following study supplies such a method.

We first consider the interior bays of many spans, alternately long and short, and then show how to use the results in other practical cases.

*Investigation.* Consider a series of spans alternately $l_1$ and $l_2$ in length, loaded respectively to $W_1$ and $W_2$. Consider

the case of uniform distribution of load, and assume as before

(1) $I$ constant, therefore $B_1 = B_2 \cdot \frac{l_2}{l_1}$.

(2) Load per ft. constant, therefore

$$\frac{W_1}{W_2} = \frac{l_1}{l_2}$$

whence $$W_1 l_1 = W_2 l_2 \left(\frac{l_1}{l_2}\right)^2$$

Referring to *Reinforced Concrete Design*, Vol. I, Appendix I, 1, p. 265, the top equations, applied to notation of Fig. 60 (*a*), and remembering that from symmetry $\alpha_2 = -\alpha_1$ and the moments at supports are equal,

$$M = -\frac{W_2 l_2}{12} - 2EB_2\alpha_1 \text{ (applied to span 1)}$$

$$= -\frac{W_1 l_1}{12} - 2EB_1\alpha_2 \text{ (applied to span 2)}$$

$$= -\frac{W_1 l_1}{12} + 2EB_1\alpha_1 \text{ since } \alpha_2 = -\alpha_1$$

Putting $$B_1 = B_2 \cdot \frac{l_2}{l_1}$$

$$-\frac{W_2 l_2}{12} - 2EB_2\alpha_1 = -\frac{W_1 l_1}{12} + 2EB_2\alpha_1 \cdot \frac{l_2}{l_1}$$

$$2EB_2\alpha_1\left(\frac{l_2}{l_1} + 1\right) = \frac{W_1 l_1}{12} - \frac{W_2 l_2}{12}$$

$$\therefore\ 2EB_2\alpha_1 = \frac{1}{12}(W_1 l_1 - W_2 l_2)\frac{l_1}{l_1 + l_2}$$

$$\therefore\ M = -\frac{W_2 l_2}{12} - \frac{1}{12}(W_1 l_1 - W_2 l_2)\frac{l_1}{l_1 + l_2} \quad . \quad . \quad (7)$$

*Case* (*a*). ALL BAYS LOADED.

$$W_1 l_1 = W_2 l_2 \cdot \left(\frac{l_1}{l_2}\right)^2$$

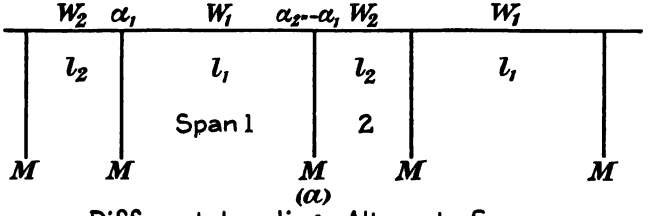
This reduces to $$M = -\frac{W_2 l_2}{12}\left[1 - \frac{l_1}{l_2} + \left(\frac{l_1}{l_2}\right)^2\right] \quad . \quad . \quad . \quad (8)$$

$$= -\frac{W_2 l_2}{12} \cdot F_1$$

*Case* (*b*). WHEN LONG SPANS ONLY ARE LOADED, $W_1 l_1 = 0$ and (7) reduces to

$$M = -\frac{W_2 l_2}{12}\left(\frac{l_2}{l_2 + l_1}\right) \quad . \quad . \quad . \quad . \quad . \quad (9)$$

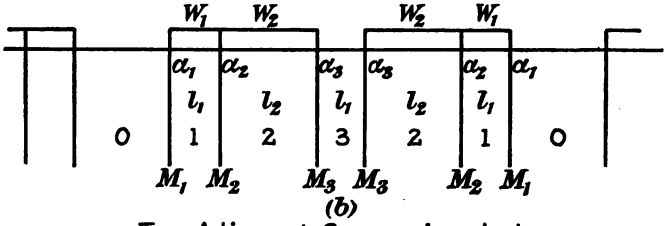
$$= -\frac{W_2 l_2}{12} \cdot F_2$$




(*a*)

Different Loadings Alternate Spans
(for Cases *a*, *b* and *c*.)

$\alpha$ = Slope. $l$ = Span. $W$ = Load. $M$ = Moment.




(*b*)

Two Adjacent Spans Loaded
(Case for Worst Moment at Support.)

Fig. 60. Many Unequal Spans Variously Loaded.

*Case* (*c*). WHEN SHORT SPANS ONLY ARE LOADED, $W_2 l_2 = 0$ and (7) reduces to

$$M = -\frac{W_1 l_1}{12} \cdot \frac{l_1}{l_2 + l_1}$$

$$= -\frac{W_2 l_2}{12} \cdot \left(\frac{l_1}{l_2}\right)^2 \cdot \frac{l_1}{l_2 + l_1} \quad . \quad . \quad . \quad (10)$$

$$= -\frac{W_2 l_2}{12} \cdot F_3$$

Notice, we obtain the same factors as for two spans, values of which are already given in Table I and Fig. 56.

The coefficient of $W_2 l_2$ is that of the moment at the end of a beam fixed horizontally both ends and uniformly loaded, and for other distributions of loading the only modification is to substitute the appropriate coefficient.

Thus, for central point load,

$$M = -\frac{Wl}{8} F_1 \text{ (case } a\text{)}$$

$$-\frac{Wl}{8} F_2 \text{ (case } b\text{)}$$

$$-\frac{Wl}{8} F_3 \text{ (case } c\text{)}$$

and for two point loads at third points

$$-\frac{Wl}{9} F_1 \text{ (case } a\text{)}$$

and so on.

The way factors $F_1$, $F_2$, $F_3$ recur in all cases indicates their general usefulness.

We have still to consider the case of two adjacent bays loaded, as in Fig. 60 (*b*), and intermediate ones unloaded.

In this case, it is clear that there are three different moments and slopes as indicated.

The case is the one giving maximum moment at the support.

Unfortunately the investigation is rather long and troublesome, but the factors are quite simple, and can afterwards be used by simple inspection from curves and tables.

From Appendix I, 1, *Reinforced Concrete Design*, Vol. 1, p. 265, we obtain two formulae for the moments at the supports. Applying these successively to different spans, we get (see Fig. 60 (*b*)):

From span 1 $M_1 = -\frac{W_1 l_1}{12} - 4EB_1 a_1 - 2EB_1 a_2$ . . . (3)

,, $M_2 = -\frac{W_1 l_1}{12} + 2EB_1 a_1 + 4EB_1 a_2$ . . . (4)

,, 2 $M_2 = -\frac{W_2 l_2}{12} - 4EB_2 a_2 - 2EB_2 a_3$ . . . (5)

,, $M_3 = -\frac{W_2 l_2}{12} + 2EB_2 a_2 + 4EB_2 a_3$ . . . (6)

From span 3 $M_3 = 0 - 4EB_1a_3 + 2EB_1a_3 = -2EB_1a_3$ (7)

„ 0 $M_1 = 0 + 4EB_2a_1 - 2EB_2a_1 = 2EB_2a_1$ .. (8)

Six equations, to solve six unknowns,

$$a_1\ a_2\ a_3\ M_1\ M_2\ M_3$$

Substituting (7) in (6),

$$\frac{W_2l_2}{12} = 2EB_2a_2 + 4EB_2a_3 + 2EB_1a_3 \quad . \quad . \quad . \quad (9)$$

Substituting (8) in (3),

$$\frac{W_1l_1}{12} = -4EB_1a_1 - 2EB_1a_2 - 2EB_2a_1 \quad . \quad . \quad (10)$$

Put $\frac{l_1}{l_2} = K$. Then $B_1 = \frac{B_2}{K}$

$$W_1l_1 = K^2W_2l_2$$

Write $B_2$ as $B$, $W_2l_2$ as $Wl$

Substituting, and rewriting equations

$$M_2 = -K^2\frac{Wl}{12} + \frac{2EBa_1}{K} + \frac{4EBa_2}{K} \quad . \quad . \quad (4')$$

$$M_2 = -\frac{Wl}{12} - 4EBa_2 - 2EBa_3 \quad . \quad . \quad . \quad . \quad (5')$$

$$\frac{Wl}{12} = 2EBa_2 + 4EBa_3 + \frac{2EBa_3}{K} \quad . \quad . \quad . \quad (9a)$$

$$K^2 \cdot \frac{Wl}{12} = -\frac{4EBa_1}{K} - \frac{2EBa_2}{K} - 2EBa_1 \quad . \quad . \quad . (10a)$$

Subtract (5′) from (4′),

$$\frac{Wl}{12}(K^2 - 1) = \frac{2EBa_1}{K} + \frac{4EBa_2}{K} + 4EBa_2 + 2EBa_3 \quad (11)$$

Rewriting $$\frac{Wl}{2EB \,.\, 12} = a_2 + \frac{2K+1}{K} \cdot a_3 \quad . \quad . \quad . \quad . \quad (9b)$$

$$-\frac{K^3}{2EB} \cdot \frac{Wl}{12} = (K+2)\,a_1 + a_2 \quad . \quad . \quad . \quad . \quad (10b)$$

$$\frac{K}{2EB} \cdot \frac{Wl}{12} \cdot (K^2 - 1) = a_1 + 2\,(K+1)\,a_2 + K \,.\, a_3 \quad (11b)$$

To eliminate $a_1$

Multiply (11$b$) by $-(K+2)$ and add to (10$b$)

$$-\frac{Wl}{12}\left\{\frac{K^3}{2EB} + \frac{K}{2EB} \cdot (K^2 - 1)\,(K+2)\right\}$$
$$= a_2\,\{1 - 2\,(K+1)\,(K+2)\} - K\,(K+2)\,a_3 \quad . \quad (12)$$

To eliminate $a_2$

Multiply (9*b*) by $-\{1-2(K+1)(K+2)\}$ and add to (12)

$$\frac{Wl}{24EB}[K^3+1+(K+2)(K^3-K-2K-2)]$$

$$=a_3\left[K(K+2)+\frac{2K+1}{K}\{1-2(K+1)(K+2)\}\right]$$

$$a_3=\frac{Wl}{24EB}\cdot\frac{(K^4+3K^3-3K^2-8K-3)K}{-3(K^3+4K^2+4K+1)} \quad . \quad (13)$$

Hence from (7)

$$M_3=-\frac{Wl}{12}\cdot\frac{1}{K}\cdot\frac{(K^4+3K^3-3K^2-8K-3)K}{-3(K^3+4K^2+4K+1)} \quad (14)$$

$$=-\frac{Wl}{12}\cdot F_6$$

where $$F_6=\frac{K^4+3K^3-3K^2-8K-3}{-3(K^3+4K^2+4K+1)}$$

Note, as a check, that when $K=1$ (spans equal) this resolves to $M_3=-\frac{Wl}{36}$, which we know to be correct (see Fig. 20).

From (9*b*),

$$a_2=\frac{Wl}{24EB}-\frac{2K+1}{K}\cdot a_3$$

$$=\frac{Wl}{24EB}+\frac{Wl}{24EB}\cdot\frac{2K+1}{K}\cdot\frac{(K^4+3K^3-3K^2-8K-3)K}{3(K^3+4K^2+4K+1)}$$

Multiplying out

$$=\frac{Wl}{24EB}\cdot\frac{(2K^4+7K^3-7K-2)K}{3(K^3+4K^2+4K+1)} \quad . \quad . \quad (15)$$

From (5), $$M_2=-\frac{Wl}{12}-4EBa_2-2EBa_3$$

Inserting values for $a_2$ and $a_3$ from (13) and (15) and multiplying out and collecting terms

$$M_2=-\frac{Wl}{12}\cdot\frac{3K^5+11K^4+6K^3+6K^2+11K+3}{3(K^3+4K^2+4K+1)} \quad (16)$$

$$=-\frac{Wl}{12}\cdot F_5$$

where $F_5$ is the value of the fraction. Note as a check, when $K = 1$ (all spans equal) this reduces to $-\frac{Wl}{9}$, which we know to be correct (see Fig. 20).

From (10*b*),

$$a_1 = -\left(\frac{K^3}{24EB} \cdot \frac{Wl}{(K+2)} + \frac{a_3}{K+2}\right)$$

Substituting value of $a_3$ from (13), multiplying out and collecting terms

$$a_1 = -\frac{Wl}{24EB} \cdot \frac{(3K^4 + 8K^3 + 3K^2 - 3K - 1)\,K}{3\,(K^3 + 4K^2 + 4K + 1)} \quad (17)$$

$$\therefore M_1 = -\frac{Wl}{12} \cdot \frac{3K^4 + 8K^3 + 3K^2 - 3K - 1}{3\,(K^3 + 4K^2 + 4K + 1)} \quad (18)$$

$$= -\frac{Wl}{12} \cdot F_4$$

where $F_4$ has the value of the fraction. Note as a check, that when $K = 1$, this reduces to $-\frac{Wl}{36}$ as it should.

All this then boils down to

$$M_1 = -\frac{Wl}{12} \cdot F_4$$

$$M_2 = -\frac{Wl}{12} \cdot F_5$$

$$M_3 = -\frac{Wl}{12} \cdot F_6$$

where $F_4$, $F_5$ and $F_6$ are expressions in terms of $K$ or $\frac{l_1}{l_2}$.

These are evaluated in Table II, and plotted in curves on Fig. 61.

For other distributions of load, it will be found that the basic formulae vary only as regards the coefficient of $Wl$, and that exactly the same formulae and factors are produced, with slightly different coefficients of $Wl$, which may all be grouped under the following general rule.

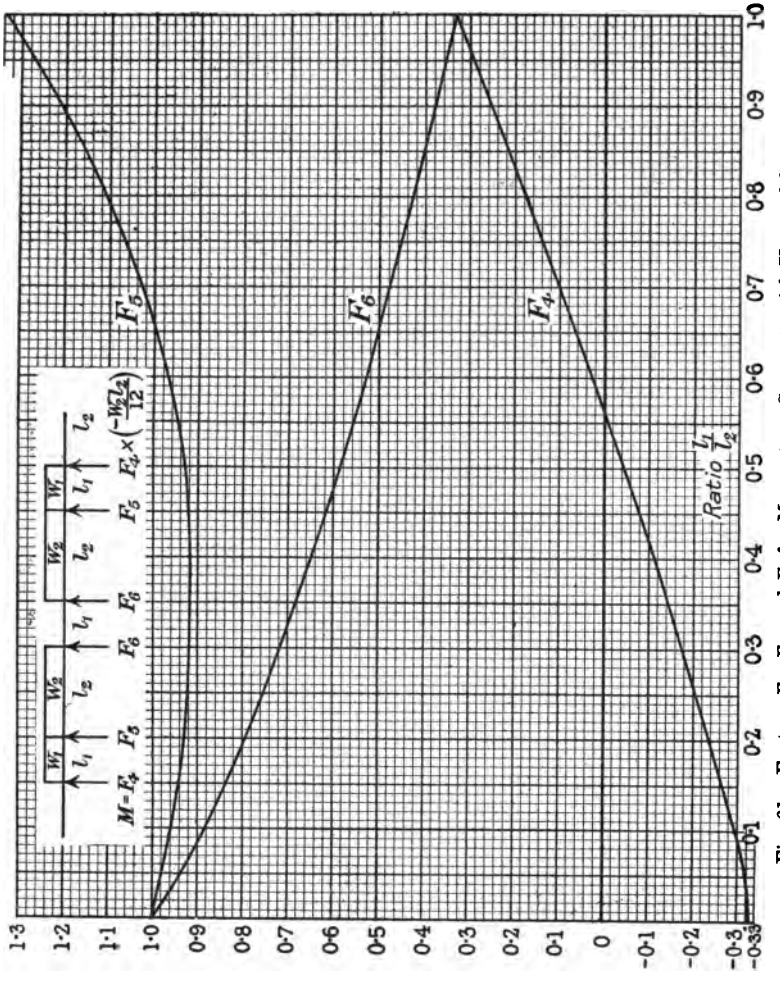



Fig. 61. Factors $F_4$, $F_5$, and $F_6$ for Moments at Supports with Unequal Spans.

**General Rule for many spans, alternately long and short, any distribution of loading.**

*Moments at supports*:

$M = M_f \,.\, F_1$ *case* (*a*) *all bays loaded,*
$= M_f \,.\, F_2$ *case* (*b*) *long bays only loaded,*
$= M_f \,.\, F_3$ *case* (*c*) *short bays only loaded.*

*Case* (*d*), *two adjacent bays loaded* (*see Fig.* 60 (*b*)):

$$M_1 = M_f \,.\, F_4$$
$$M_2 = M_f \,.\, F_5$$
$$M_3 = M_f \,.\, F_6$$

*$M_f$ for any distribution of loading is the fixing moment at the end of a beam fixed both ends and loaded with that distribution and has following values:*

*Uniform load* $M_f = -\frac{Wl}{12}$

*Point load mid-span* $M_f = -\frac{Wl}{8}$

2 *Point loads at third points* $M_f = -\frac{Wl}{9}$

TABLE II. Factors $F_4$, $F_5$ and $F_6$.

| Ratio of spans | $F_4$ | $F_5$ | $F_6$ |
|---|---|---|---|
| 1·0 | ·33 | 1·33 | ·33 |
| ·9 | ·255 | 1·21 | ·38 |
| ·8 | ·176 | 1·11 | ·43 |
| ·7 | ·1 | 1·03 | ·48 |
| ·6 | ·0285 | ·97 | ·53 |
| ·5 | −·0428 | ·93 | ·59 |
| ·4 | −·1116 | ·92 | ·65 |
| ·3 | −·175 | ·92 | ·72 |
| ·2 | −·236 | ·93 | ·795 |
| ·1 | −·295 | ·96 | ·885 |
| 0 | −·333 | 1·0 | 1·0 |

The negative value of $F_4$ for ratios of spans less than ·5 means that the moment at the support is then positive.

*Example.* As an example showing the use of these factors, take the specific case of many spans alternately long and short, in which the ratio

$$K = \frac{l_1}{l_2} = \tfrac{1}{2}$$

Then we can write down the moments in all cases by simple inspection from Tables I and II of the Factors 1 to 6.

UNIVERSITY OF CALIFORNIA
DEPARTMENT OF CIVIL ENGINEERING
BERKELEY, CALIFORNIA

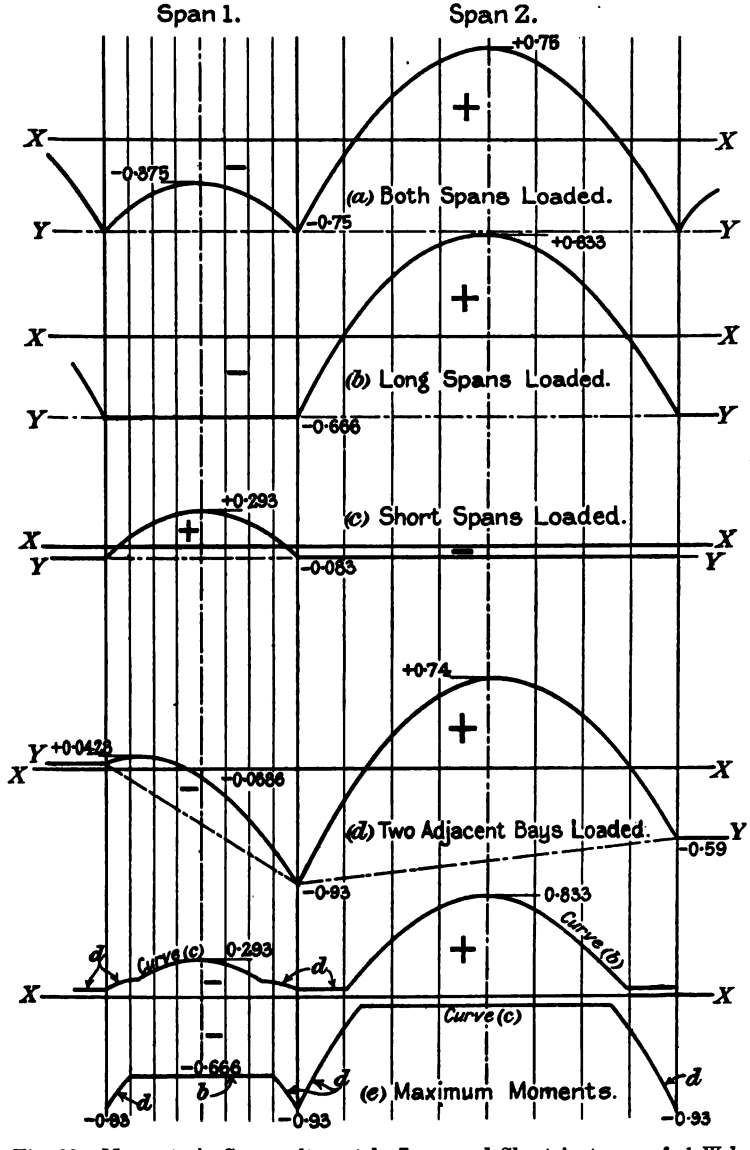



Fig. 62. Moments in Spans alternately Long and Short in terms of $\frac{1}{12} W_2 l_2$ for Ratio of Spans 1 : 2. Uniform Distribution.

*Case* (*a*). BOTH SPANS LOADED.

$$M \text{ at support} = -\frac{Wl}{12} \cdot F_1$$
$$= -\cdot 75 \times \frac{Wl}{12}$$

where $W$ and $l$ refer to *long* span. In Fig. 62 (*a*) we set down $YY$ at ·75 below $XX$ and on the two spans erect parabolas of height $\frac{W_2 l_2}{8}$ on the long span

$$\left[= 1{\cdot}5 \times \frac{Wl}{12}\right] \text{ and } \frac{W_1 l_1}{8} = \frac{W_2 l_2}{4 \times 8} = \cdot 375 \cdot \frac{W_2 l_2}{12}$$

on the short span. (All figures kept in terms of $\frac{W_2 l_2}{12}$ for convenience.)

*Case* (*b*). LONG SPANS ONLY LOADED.

$$M \text{ at support} = -F_2 \cdot \frac{Wl}{12}$$
$$F_2 \text{ from table} = \cdot 666$$

In Fig. 62 (*b*) set down $YY$ at ·666 below $XX$, and on the long spans only erect a parabola of $\left[1{\cdot}5 \times \frac{Wl}{12}\right]$.

*Case* (*c*). SHORT SPANS ONLY LOADED.

$$M \text{ at support} = -F_3 \cdot \frac{W_2 l_2}{12}$$
$$F_3 = \cdot 083 \text{ from table}$$

In Fig. 62 (*c*) set down $YY$ at ·083 below $XX$, and on the short spans only erect a parabola of height

$$\cdot 375 \left(\times \frac{W_2 l_2}{12}\right)$$

*Case* (*d*). TWO ADJACENT BAYS LOADED.

On the left-hand support (between the loaded short span, and the unloaded long span) the moment is

$$M_1 = F_4 \times -\frac{W_2 l_2}{12}$$

$F_4$ from the table is $-$ ·0428.

Hence we erect ·0428 *above* $XX$ in Fig. 62 (*d*). At the support between loaded spans

$$M_2 = F_5 \times -\frac{W_2 l_2}{12}$$

$F_5$ from table is ·93.

Set down ·93 below $XX$ in Fig. 62 (*d*). At the support between loaded long span and unloaded short span, the moment is

$$M_3 = F_6 \times -\frac{W_2 l_2}{12}$$

$F_6$ from table is ·59.

On the dotted line $YY$ joining these points erect skew parabolas of same heights as before (see p. 7).

On the unloaded panels, at the sides, the moment curves are horizontal lines as drawn.

Fig. 62 (*e*) is a curve which combines at each section the worst moment selected from (*a*), (*b*), (*c*) or (*d*).

In regard to (*d*) it must be remembered that any of the supports might be the one between the two loaded panels, and curve (*e*) is drawn with this negative cusp at each support accordingly.

In practice, curves (*a*), (*b*), (*c*) and (*d*) would not be drawn separately, but would be superimposed on the same curve, and would so form curve (*e*) direct without further labour.

**Treatment of End Spans.** The use of these factors lends itself to the determination of moments at the supports of interior spans of unequal lengths with sufficient accuracy even when the spans are not alternately long and short as will be shown.

Before doing so, it will be well to explain how end bays should be treated. It will be remembered that for two spans we applied factors $F_1$, $F_2$ and $F_3$ to $\frac{W_2 l_2}{8}$ to obtain the moment at the interior support in cases (*a*), (*b*) and (*c*) respectively. For interior supports, we apply them to the moment $\frac{W_2 l_2}{12}$.

For end bays, to obtain the moment at the support first removed from the wall, the factors should be applied to $\frac{W_2 l_2}{10}$, the intermediate value.

As a check on this, consider the special case of all spans equal.

(*a*) All spans loaded.

Moment at support nearest wall

$$M = F_1 \times \frac{Wl}{10} = \frac{Wl}{10}$$

This agrees with the value we adopted for the support nearest the wall, see Fig. 28 (which we obtained by considering three spans).

(*b*) Alternate bays loaded.

$$M = F_2 \times \frac{Wl}{10} \text{ or } F_3 \times \frac{Wl}{10} = \frac{Wl}{20}$$

This also agrees with the values we adopted for end spans all equal.

When we come to Factors 4, 5 and 6 (two adjacent spans loaded, next unloaded), we multiply our factors by $\frac{Wl}{11}$.

As a check, consider case of all equal spans, two end ones loaded, next one unloaded, and so on.

Then moment at support nearest wall (i.e. one span from the outer support).

$$M = F_5 \cdot \frac{Wl}{11} = \cdot 121 Wl$$

[since $F_5 = 1 \cdot 33$]

Now the moment here is as follows (calculated accurately):

| | | |
|---|---|---|
| 3 | spans | $\cdot 116\, Wl$ |
| 4 | ,, | $\cdot 120$ |
| 5 | ,, | $\cdot 119$ |

The result is therefore a very close approximation.

**General Rule. To obtain the moment at the support one span from the outer support (where the beam is free) calculate the moment by the use of the factors as before, and then increase in the ratio $\frac{12}{10}$ when $F_1$, $F_2$ and $F_3$ are used, and in the ratio $\frac{12}{11}$ when $F_5$ is used.**

## SUMMARY.

The moments at supports are therefore given by multiplying the appropriate factors $F_1$ to $F_6$ as given on p. 111 by the following moment.

| Load Distribution | Two spans | Many spans | | |
|---|---|---|---|---|
| | | Interior supports | Support one span from end | |
| | | | All or alternate spans loaded | Adjacent spans loaded |
| 1 | 2 | 3 | 4 | 5 |
| Uniform | $\frac{Wl}{8}$ | $\frac{Wl}{12}$ | $\frac{Wl}{10}$ | $\frac{Wl}{11}$ |
| Point load at mid-span | ·187 $Wl$ | ·125 $Wl$ | ·15 $Wl$ | ·136 $Wl$ |
| 2 point loads at third points | ·167 $Wl$ | ·111 $Wl$ | ·134 $Wl$ | ·121 $Wl$ |

$Wl$ applies to the longer span of the two between which lies the support under consideration.

**Case of all Unequal Spans.** The factors have been calculated for spans alternately long and short. In practice the spans are often all unequal, without this regular alternation.

It will be shown by a practical example, which will then be calculated accurately by the theorem of three moments, that if the factors are applied successively to each support, only the spans immediately on either side of this support being considered, the results are quite accurate enough for all practical purposes.

The author believes this to be the first and only practical method of solving many, all unequal, spans in a practical manner, and the work on these Factors which has led up to this and through which the reader may or may not have patiently waded, is fully justified by this wide application.

**Example of all Unequal Spans.** We will now show how a practical case of all unequal spans may be treated by the use of the author's factors $F_1$ to $F_6$ (see Fig. 63).

Let the successive spans be 10 ft., 15 ft., 20 ft. and 10 ft., the two ends free, the loading one ton per ft. run and all live, so that any combination of loaded spans may occur.

We may neglect the case of all spans loaded. This is only required for dead load. The following cases always give worse moments for live load.

*Case* (*a*). Spans 2 and 4 loaded, 3 and 1 unloaded.

Referring to diagram (Fig. 63) we have the moment at the first interior support $M_1 = F_2 \times -\frac{W_2 l_2}{10}$ *

Ratio of spans $= \frac{10}{15} = \cdot 66$ (we consider spans 1 and 2 only)

$\therefore F_2 = \cdot 6$ [from Fig. 56]

$\therefore M_1 = -\cdot 6 \times \frac{15^T \times 15 \text{ ft.}}{10} = -13 \cdot 5 \text{ ft. tons}$

Similarly $M_2 = F_3 \times -\frac{W_3 l_3}{12}$ †

Ratio of spans $\frac{15}{20} = \cdot 75$

$\therefore F_3 = \cdot 245$ [from Fig. 56]

$\therefore M_2 = \cdot 245 \times \frac{20^T \times 20 \text{ ft.}}{12} = -8 \cdot 2 \text{ ft. tons}$

Similarly $M_3 = F_3 \times -\frac{W_3 l_3}{10}$

$= -\cdot 083 \times \frac{20^T \times 20 \text{ ft.}}{10} = -3 \cdot 34 \text{ ft. tons}$

Knowing these moments, it is easy to draw the moment diagram as in Fig. 63, Curve (*a*).

* $F_2$ because the short span is unloaded and the long one loaded (see p. 111).
$W_2 l_2$ because we always put the moment in terms of the *longer* span.
10 because the support is that nearest the wall support (see p. 116, col. 4).

† $F_3$ because the long span is unloaded and the short one loaded (see p. 111).
$W_3 l_3$ because we always take the longer span.
12 because the support is not a support nearest the wall (see p. 116, col. 3).

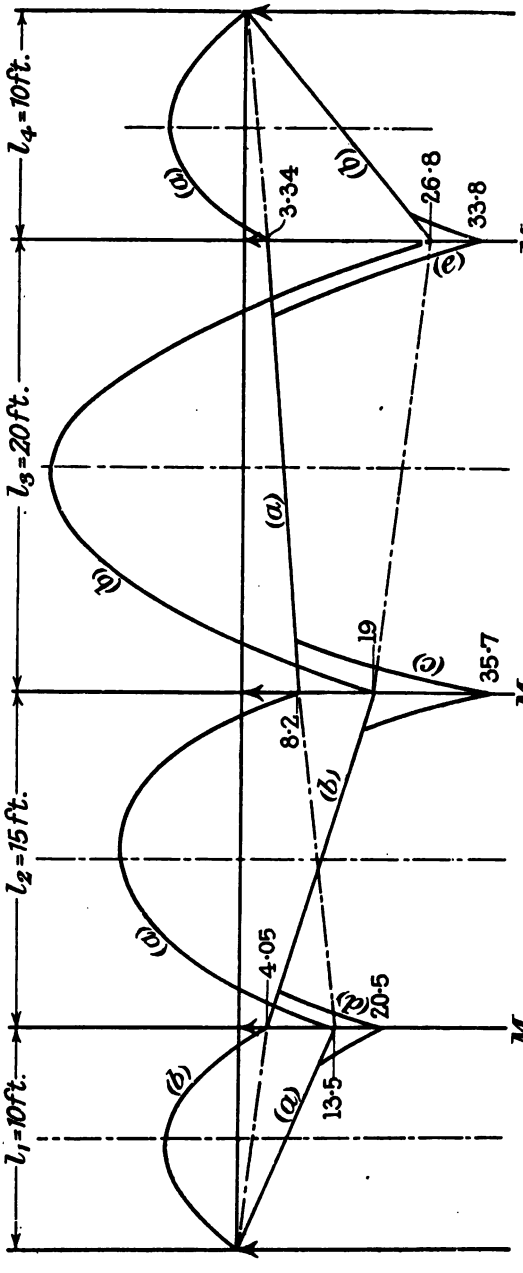



Fig. 63. Curve of Maximum Moments. Four Unequal Spans. [Figures are Moments in foot tons.]

*Case* (*b*). Spans 1 and 3 loaded, 2 and 4 unloaded.

$$M_1 = F_3 \times \frac{W_2 l_2}{10}$$

$$= \cdot 18 \times \frac{15 \times 15}{10} = -4 \cdot 05 \text{ ft. tons}$$

$$M_2 = F_2 \times \frac{W_3 l_3}{12}$$

$$= \cdot 57 \times \frac{20 \times 20}{12} = -19 \text{ ft. tons}$$

$$M_3 = F_2 \times \frac{W_3 l_3}{10}$$

$$= \cdot 67 \times \frac{20 \times 20}{10} = -26 \cdot 8 \text{ ft. tons}$$

[Compare the labour of this with that of a solution by the theorem of three moments.]

We can now easily draw the moment diagram in Fig. 63 (*b*).

NOTE. Cases (*a*) and (*b*) give worst positive and negative moments towards midspan, as will be seen on Fig. 63.

*Case* (*c*). Spans 1 and 4 unloaded, 2 and 3 loaded.

This gives maximum moment at $M_2$.

Ratio of spans $K = \frac{l_2}{l_3} = \frac{15}{20} = \cdot 75$

$$\therefore F_5 = 1 \cdot 07$$

$$\therefore M_2 = F_5 \times -\frac{W_3 l_3}{12}^*$$

$$= -1 \cdot 07 \times \frac{20 \times 20}{12} = -35 \cdot 7 \text{ ft. tons}$$

Now, strictly speaking, to draw the curve we need to calculate $M_1$ and $M_3$ under the condition of loading we are considering, as we did in Fig. 62 (*d*).

Thus $$M_1 = F_4 \times -\frac{W_3 l_3}{12}$$

and $$M_3 = F_6 \times -\frac{W_3 l_3}{12}.$$

* $F_5$ because we are considering the moment between two adjacent loaded spans (see p. 111 and Fig. 60 *b*).

$W_3 l_3$ because we consider the longer of the two adjacent spans.

12 because it is an interior support (col. 3, p. 116).

But as it is only the cusp we want to use, an experienced designer may obviate this after having done many examples fully, and draw the cusp in by eye. It is generally roughly parallel to the parabola drawn for case (*a*) or (*b*).

*Case* (*d*). Spans 1, 2 and 4 loaded, 3 unloaded. This gives negative cusp at $M_1$.

$$M_2 = F_5 \times -\frac{W_2 l_2}{10}$$

$$= -1{\cdot}0 \times \frac{15 \times 15}{11} = -20{\cdot}5 \text{ ft. tons}$$

*Case* (*e*). Spans 1, 3 and 4 loaded, 2 unloaded. This gives negative cusp at $M_3$.

$$M_3 = F_5 \times -\frac{W_3 l_3}{11}^{*}$$

$$= -{\cdot}93 \times \frac{20 \times 20}{11} = -33{\cdot}8 \text{ ft. tons}$$

The process then resolves itself into calculating moments at each support for the two cases of alternate bays loaded (there are only two such cases, however many spans) and the negative cusp at each support when the two adjacent bays are loaded.

In Fig. 63, curves (*a*) and (*b*) are the moments with alternate bays loaded, drawn superimposed, and the cusps are added as described (curves *c*, *d* and *e*).

The result is the curve of maximum live load moments at all sections.

In cases where the dead load is an appreciable proportion, the dead load curve can just as easily be drawn, using Factor $F_1$ at each support, and combined with the live load curves as was done in the earlier chapters.

We will now check our results by the theorem of three moments. The method is so tedious even for four spans that we will confine ourselves to case (*a*), spans 2 and 4 loaded, although there are really five cases (*a*) to (*e*) requiring solution, as above.

* $F_5$ because we are considering moment between two adjacent loaded spans (see p. 111 and Fig. 60 *b*).

$W_3 l_3$ because these refer to the longer of the adjacent spans.

11 because $M_3$ is at a support one span from end (see col. 5, p. 116).

We have $S_1 = 0$

$$S_2 = \frac{15^T \times 15 \text{ ft.}}{8} \times 15 \text{ ft.} \times \tfrac{2}{3}$$
$$= 281$$
$$S_3 = 0$$
$$S_4 = \frac{10^T \times 10 \text{ ft.}}{8} \times 10 \text{ ft.} \times \tfrac{2}{3}$$
$$= 83{\cdot}3$$

$\frac{y}{l} = \frac{1}{2}$ in all cases.

Applying theorem to spans 1 and 2 (see p. 84),

$$2M_1(10 + 15) + M_2 \times 15 = 3 \times 281$$
$$50M_1 + 15M_2 = 843$$
$$10M_1 + 3M_2 = 168{\cdot}6 \quad . \quad . \quad . \quad (1)$$

Applying it to spans 2 and 3,

$$M_1 \times 15 + 2M_2(15 + 20) + M_3 \times 20 = 3 \times 281$$
$$3M_1 + 14M_2 + 4M_3 = 168{\cdot}6 \quad . \quad (2)$$

Applying it to spans 3 and 4,

$$M_2 \times 20 + 2M_3(20 + 10) = 3 \times 83{\cdot}3$$
$$M_2 + 3M_3 = 12{\cdot}5 \quad . \quad . \quad . \quad . \quad (3)$$

From (3), $4M_3 = 16{\cdot}6 - \frac{4}{3}M_2$

Substituting in (2),

$$3M_1 + 12\tfrac{2}{3}M_2 = 152 \quad . \quad . \quad . \quad . \quad (4)$$

Multiply by 10,

$$30M_1 + 126{\cdot}6M_2 = 1520$$

Multiply (1) by 3,

$$30M_1 + 9M_2 = 505{\cdot}8$$
$$117{\cdot}6M_2 = 1014{\cdot}2$$
$$M_2 = 8{\cdot}64$$

Substituting in (1), $10M_1 = 168{\cdot}6 - 25{\cdot}92$

$$= 142{\cdot}7$$
$$M_1 = 14{\cdot}27$$

Substituting in (3), $3M_3 = 12{\cdot}5 - 8{\cdot}64$

$$= 3{\cdot}86$$

The results compare as follows:

| | By theorem of 3 moments | By use of Factors |
|---|---|---|
| $M_1$ | 14·27 | 13·5 ft. tons |
| $M_2$ | 8·64 | 8·2 |
| $M_3$ | 3·86 | 3·34 |

The theory of three moments is, of course, correct, but there is no comparison in the labour required, since by the factor method the results are with a little practice simply written down.

Also, whereas the author's factor method does not increase in complexity with the number of spans, solution by the theorem of three moments does so in a very marked degree.

# PART II

## MOMENTS IN COLUMNS

### CHAPTER XI

#### OUTSIDE COLUMNS

It was explained at some length in Vol. I that when a beam deflects, the column with which it is monolithic necessarily bends also, and therefore has a bending moment in it which causes stresses which cannot be neglected.

These stresses are particularly important in the case of outside columns.

An example was given in Vol. I, p. 146 of an outside column where the stress was 300 lbs./inch$^2$, neglecting such bending, and 2200 lbs./inch$^2$ when it was taken into account.

Some so-called specialist firms design without taking these factors into account. Some so-called specialist firms also from time to time pay for structures which have unaccountably collapsed.

The author would prefer to design outside columns by eye to making misleading calculations in which the most important factor is omitted.

The author has heard people argue that if they design their beams without reckoning on the stiffness of the columns, there is no need to design columns for bending—he will refrain from quoting from works where this fallacy is supported. Unfortunately (or otherwise, the author thinks on the whole otherwise) we cannot alter the laws of nature by the simple process of making an assumption, and one of the laws of nature is that if you load an outside beam monolithic with an outside column, you bend both the beam and the column, and therefore develop bending stresses in the column. The truth is, people have neglected these stresses because they could not calculate them.

While the treatment in Vol. I, Chap. VII, is quite correct, we can make the results much easier to use in everyday practice, and will now proceed to do so.

**Outside Columns monolithic with Beams of One Span.** In Vol. I, Appendix I, 5, p. 273, it was shown that the slope induced at the end of such a column is given by

$$Ea = -\frac{Wl}{12\,(KC + 2B)} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where $a$ = the angle of slope (circular measure),
$E$ = coefficient of elasticity of concrete,
$W$ = total load on span,
$l$ = span,
$K$ = a constant depending on conditions of fixity at the bottom of the column, being the numerical factor in the last column but one of the table on page 125, Vol. I, and varying from 2 to 6, but being 4 for bottom fixed in direction,
$C$ = stiffness of column, given by the ratio

$$\frac{\text{moment of inertia of column}}{\text{length of column}},$$

$B$ = stiffness of beam, given by the ratio

$$\frac{\text{moment of inertia of beam}}{\text{length of beam}}.$$

The moment in the column is

$$M = KCEa = -\frac{KC \times Wl}{12\,(KC + 2B)}$$
$$= -\frac{Wl}{12}\left(\frac{KC}{KC + 2B}\right) \quad . \quad . \quad . \quad (2)$$
$$= -\frac{Wl}{12} \cdot F_{14}.$$

if we denote by $F_{14}$ the factor $\frac{KC}{KC + 2B}$.

This factor it is convenient to evaluate and to express in tabular form and in the form of a curve as in Fig. 64.

The curve in Fig. 64 shows a break. This is only because it is convenient to change the scale to get a great range of values without loss of accuracy, and there is, of course, no break in the relationship between $F_{14}$ and $\frac{KC}{B}$.

Fig. 65 is an enlarged part of Fig. 64 for values of $\frac{KC}{B}$ up to 1.

Those unacquainted with this matter think there is a complication in working these things out. But with the matter as herein presented it is quite simple and rapid.

TABLE III.

Values of $F_{14}$ for determining moment in outside column with beam of one span.

| Ratio $\frac{KC}{B}$ | $\frac{B}{KC}$ | $F_{14}$ |
|---|---|---|
| ·1 | 10 | ·048 |
| ·2 | 5 | ·09 |
| ·5 | 2 | ·20 |
| 1 | 1 | ·33 |
| 2 | ·5 | ·5 |
| 5 | ·2 | ·715 |
| 10 | ·1 | ·835 |
| 20 | ·05 | ·912 |
| 30 | ·033 | ·935 |
| ∞ | 0 | 1·0 |

In a later chapter are given tables of the author's standard beams and columns and their properties, from which the values of $C$ and $B$ can be found at once. $K$ is 4 for single story outside columns and 6 for upper story outside columns.

*Example* 1 (top floor of a building).

$W$ (load) = 25,000 lbs.

$l$ (span) = 25 ft. = 300 ins.

height of col. = 12½ ft. = 150 ins.

approximate moment in beam $= \frac{Wl}{8} = \frac{25{,}000 \times 300}{8}$

$= 938{,}000$ in. lbs.*

Modulus required (in steel units)

$$= \frac{938{,}000}{16{,}000} = 58{\cdot}6$$

Clearly Standard Beam 5 is required (see Table V, p. 156)

* Really slightly less, because of slight restraint offered by columns.

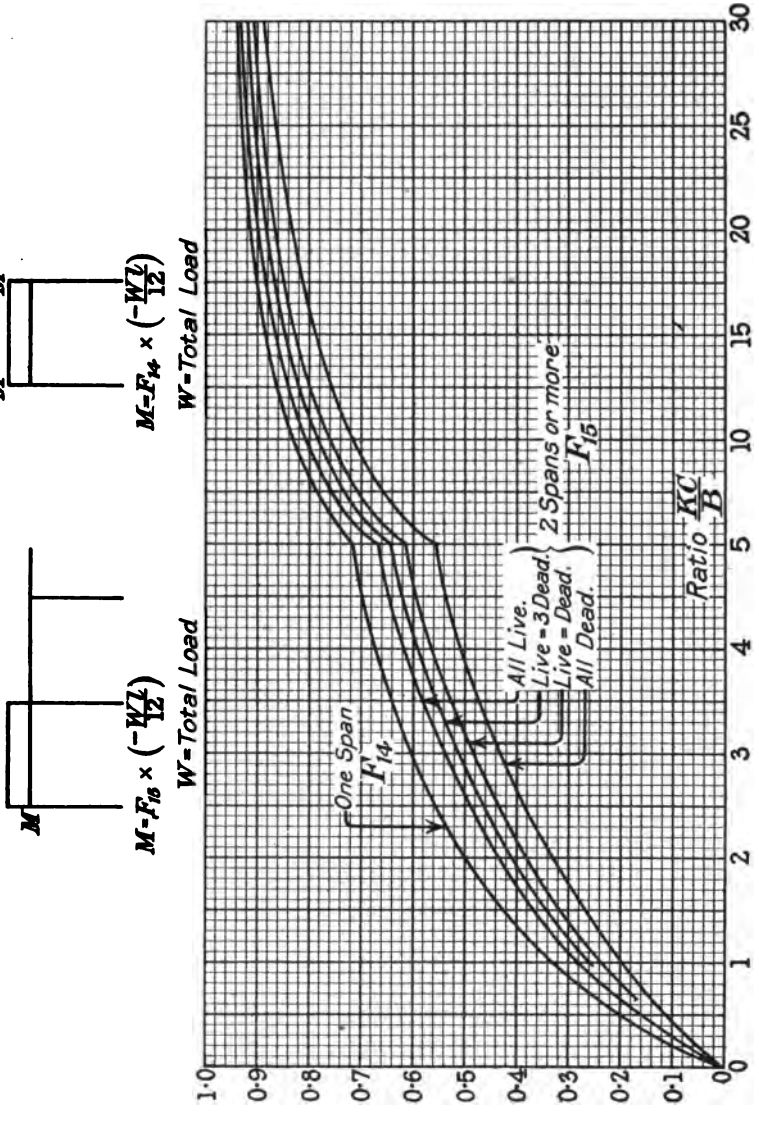



Fig. 64. Factors $F_{14}$ and $F_{15}$ for Moments in Outside Columns.

whence $I = 10{,}300$ (concrete units) and

$$B = \frac{I}{l} = \frac{10{,}300}{300} = 34{\cdot}4 \text{ ins.}^3$$

We will try standard column 24 × 6 for the outside column.

Hence from Table II, page 150,

$$I = 454{\cdot}5 \text{ ins.}^4$$

$$C = \frac{I}{l} = \frac{454{\cdot}5}{150} = 3{\cdot}03$$

$$\therefore KC = 6 \times 3{\cdot}03 = 18{\cdot}18$$

and $$\frac{KC}{B} = \frac{18{\cdot}18}{34{\cdot}4} = {\cdot}53$$

whence $$F_{14} = {\cdot}21 \text{ from Fig. 65}$$

and $$M = \frac{Wl}{12} \times {\cdot}21$$

$$= 131{,}000 \text{ in. lbs.}$$

Bending stress in column $$= \frac{My}{I} = \frac{131{,}000 \times 3''}{454{\cdot}5}$$

$$= 865$$

Direct stress in column $$\frac{R}{A} = \frac{12{,}500}{166{\cdot}4^*} = 75$$

Max. compr. stress in column ($c$) $= 940$ lbs./ins.$^2$

If this example, again, does not convince the reader of the necessity of *not* neglecting these bending stresses, nothing will.

It will be noticed that the tensile stress in the concrete by this calculation is 865 − 75 = 790 lbs. per sq. in. This is, of course, far greater than the concrete can stand, and the concrete would have cracked on the tension side. In that case, the method employed above is not strictly accurate.

It is, therefore, of interest to investigate whether a more accurate calculation in which the tensile strength of the concrete is neglected gives a very different value for $c$.

For this more accurate calculation we may employ the method of page 69, Volume I (Bending and Compression). We must however in this calculation not take the same moment in the column as above (131,000 in. lbs.) because the figure which we used for the moment of inertia of the column (454·5 ins.$^4$) from Table II, p. 150, was calculated on the basis of the whole area of concrete being stressed, and if we are to neglect the concrete in tension, this would be inaccurate in cases like the present, where tension occurs.

It is an inherent difficulty of this problem that the *exact* calculation of the moment of inertia involves a knowledge of the position of the neutral axis, which is unknown until the calculation is completed, but it will be sufficiently accurate to use for this calculation the moment of inertia about the centre line of the column, on the assumption that this is also the neutral axis, so that on

* This is the equivalent area of column 24 × 6 from Table II, p. 150.

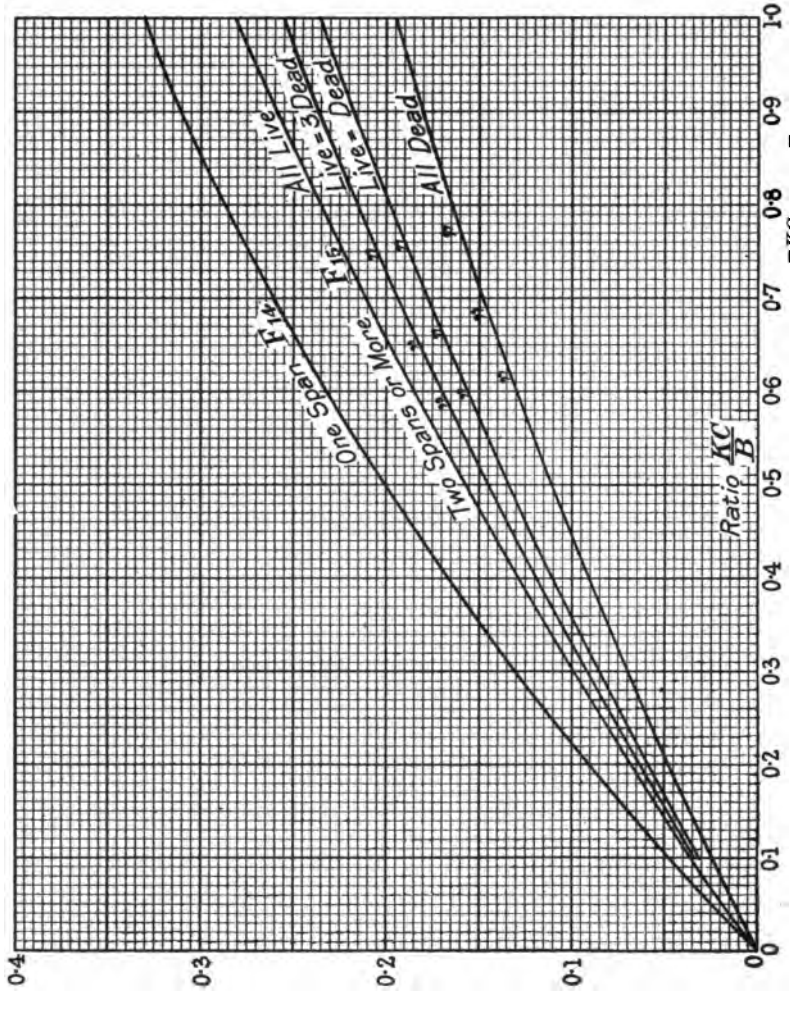



Fig. 65. Factors $F_{14}$ and $F_{15}$ for Moments in Outside Columns $\left[\frac{KC}{B} \text{ up to } 1\right]$.

the compression side we have half the column area and half the steel, and on the tension side half the steel only, the concrete on the tension side not acting.

The moment of inertia is then

due to compression concrete $\frac{24'' \times 3^3}{3} = 216$ ins.$^4$

due to compression steel $\cdot 8 \times 1^2 \times 15 = 12$ ins.$^4$

due to tension steel $\cdot 8 \times 1^2 \times 14 = 11 \cdot 2$ ins.$^4$

$239 \cdot 2$ ins.$^4$

Note in passing that this is, near enough, *one half* the value given in Table II, p. 150, and always will be so when the concrete is neglected on one side and the steel is only a small percentage.

Revising the calculations of p. 127 on this basis we have

$$I = 239 \cdot 2$$

$$KC = \frac{6 \times 239 \cdot 2}{150} = 9 \cdot 6$$

$$\frac{KC}{B} = \frac{9 \cdot 6}{34 \cdot 4} = \cdot 278$$

$$\therefore F_{11} \text{ (from Fig. 65)} = \cdot 122$$

$$\text{and } M \text{ in col.} = \frac{Wl}{12} \times \cdot 122 = 76{,}000 \text{ in. lbs.}$$

Proceeding now in accordance with method of p. 63, Vol. I, we have

$$R = 12{,}500$$

$$\text{eccentricity } e = \frac{M}{R} = \frac{76{,}000}{12{,}500} = 6 \cdot 1 \text{ ins.}$$

$$f = 1''$$

$$e + f = 7 \cdot 1''$$

$$\frac{e+f}{d} = \frac{7 \cdot 1}{4} = 1 \cdot 78$$

$$p = \frac{4 \times \cdot 2 \times 100}{4 \times 24} = \cdot 83\ \%$$

whence $t_1 = 15 \cdot 6$ (Fig. 32, Vol. I.)

whence $a_1 = \cdot 836$ (Fig. 14, Vol. I.)

and $n_1 = \cdot 49$ (Fig. 11, Vol. I.)

Then from (13), p. 67, Vol. I,

$$c = \frac{2P(e+f)}{nba} = \frac{25{,}000 \times 7 \cdot 1}{(\cdot 49 \times 4) \times 24 \times (\cdot 836 \times 4)}$$

$$= 1120$$

$$t = t_1 \times c = 15 \cdot 6 \times 1120 = 17{,}500 \text{ lbs./ins.}^2$$

It will be seen that the neutral axis is $\cdot 49 \times 4 = 2''$ from the compressed edge, so that actually less of the concrete is carrying stress than we assumed (3″), but this makes very little difference to the value of the moment of inertia. See the calculation for a similar case on p. 134 (footnote), and the notes in regard to the difference in stresses as between the simple and exact methods of calculation.

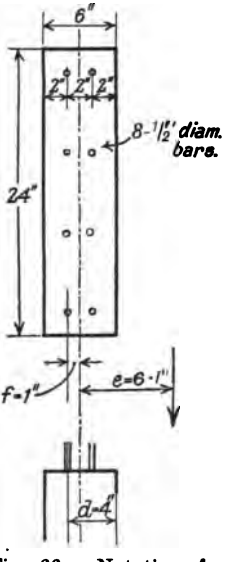



Fig. 66. Notation for finding Stresses in Outside Column, 24″ × 6″.

*Example* 2. Top floor of building.

$W$ (load) = 50,000 lbs. (twice that of last example)

$l$ (span) = 12′ 6″ = 150″ (half ,, ,, ,, ,, )

It will be found that the same beam is needed as before, and we will also use the same column.

$$C = \frac{454{\cdot}5}{300} = 1{\cdot}515$$

$$KC = 9{\cdot}09$$

$$B = \frac{10{,}300}{150} = 68{\cdot}8$$

$$\therefore \frac{KC}{B} = \frac{9{\cdot}09}{68{\cdot}8} = {\cdot}132$$

whence $F_{14}$ from Fig. 65

$$\left[\text{or thus} \quad \frac{KC}{KC + 2B} = \frac{9{\cdot}09}{9{\cdot}09 + 137{\cdot}6}\right] = {\cdot}062$$

and

$$M = -\frac{Wl}{12} \times {\cdot}062$$

$$= -38{,}800$$

and

$$\text{Bending stress} = \frac{38{,}800 \times 3}{454{\cdot}5} = 256$$

$$\text{Direct stress} = \frac{25{,}000}{166{\cdot}4} = 150$$

$$\text{Max. combined stress} = \overline{406} \text{ lbs./ins.}^2$$

Here the direct stress more nearly equals the bending stress, and the method employed is accurate, because the tension is only 106 lbs./ins.$^2$

Now many fruitless attempts have been made to simplify the matter by considering the beam to sit on the edge of the column, or some other place, and calculating moments accordingly. Let us see what the eccentricities were in the last two examples.

*Example* 1.

$$\text{Eccentricity} = \frac{\text{moment in col.}}{\text{reaction in col.}} = \frac{131{,}000}{12{,}500} = 10\tfrac{1}{2} \text{ ins.}$$

*Example* 2.

$$\text{Ecc.} = \frac{M}{R} = \frac{38{,}800}{25{,}000} = 1{\cdot}55 \text{ ins.}$$

It should be clear from this that if the same column monolithic with the same beam can in one case have an eccentricity of $10\frac{1}{2}$ ins. and in another case 1·55 ins., all such attempts are abortive.

*Example* 3. We will now take an example including lower floors (see Fig. 68). We will adopt beams 25′ span, columns 12′ 6″ long, load per beam 25,000 lbs., all as in Example 1, and as the columns 24 × 6 gave rather high stresses, and are in any case rather slender for this story height (both for buckling and wind pressure) we will calculate stresses on standard 24 × 12 columns (see p. 150) adopting standard 20 × 8 beams (T.B. 5) (see p. 156) as before.

From p. 150, $I$ of col. 24 × 12 is 4170 ins.$^4$ and equivalent area $A$ is 325 ins.$^2$ We proceed as before.

*Top tier.* $$\frac{KC}{B} = 6 \times \frac{4170}{150} \times \frac{300}{10{,}300} = 4{\cdot}87$$

$$\therefore F_{14} \text{ (from Fig. 64)} = {\cdot}71$$

$$\therefore M \text{ in col.} = {\cdot}71 \times \frac{25{,}000 \times 300''}{12}$$

$$= 444{,}000 \text{ in. lbs.}$$

$$\text{Stress in col. due to bending} = \frac{My}{I} = \frac{444{,}000 \times 6''}{4170}$$

$$= 640 \text{ lbs./ins.}^2$$

$$\text{Direct stress} = \frac{12{,}500}{325} = 38{\cdot}5 \quad ,,$$

$$\text{Max. combined stress} = 678{\cdot}5 \quad ,,$$

Clearly here is considerable tension. Let us again check the value of $c$ by making the more accurate calculation, neglecting tension resistance in the concrete.

The moment of inertia of the column about the centre line, assuming the neutral axis to lie 4″ from the compressed face (trial value, see later)

| | |
|---|---|
| compression concrete | $\frac{24 \times 6^3}{3} = 1730$ |
| compression steel | $(3 \times {\cdot}44) \times 4 \times 14 = 296$ |
| tension steel | $3 \times {\cdot}44 \times 4^3 \times 15 = 317$ |
| | 2343 ins.$^4$ |

As before, this is approximately one-half the value of $I$ as given in Table II, p. 150, when the whole area of concrete is acting (when tension is small) which was 4170.

$$\frac{KC}{B} = \frac{6 \times 2343 \times 300}{150 \times 10{,}300} = 2{\cdot}71$$

$$\therefore F_{14} \text{ (from Fig. 64)} = {\cdot}58$$

$$\therefore M \text{ in column} = {\cdot}58 \times \frac{25{,}000 \times 300}{12}$$

$$= 363{,}000 \text{ ins. lbs.}$$

$$\therefore \text{ eccentricity } e=\frac{M}{R}=\frac{363{,}000}{12{,}500}=29 \text{ ins.}$$

$$f=4''$$

$$e+f=33''$$

$$\frac{e+f}{d}=\frac{33''}{10''}=3{\cdot}3$$

$$p=\frac{3\times{\cdot}44\times100}{10\times24}={\cdot}55$$

$\therefore t_1=25{\cdot}4$ (Fig. 32, Vol. I.)

whence $n_1={\cdot}36$ (Fig. 11, Vol. I.)

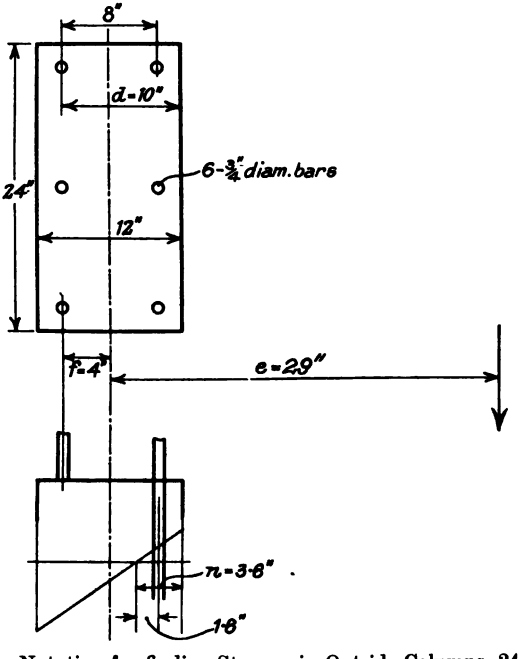



Fig. 67. Notation for finding Stresses in Outside Columns, 24" × 12".

Taking the steel into account, as in Vol. I, p. 69,

$$n={\cdot}36\times10''=3{\cdot}6''.$$

Compression from concrete

$$=\frac{nbc}{2}=\frac{3{\cdot}6\times24}{2}c=40{\cdot}8c$$

Compression from steel

$$=14\times1{\cdot}32\times\frac{1{\cdot}6}{3{\cdot}6}\times c=\ 8{\cdot}4c$$

$$\overline{49{\cdot}2c}$$

Hence equivalent width $=24\times\frac{49{\cdot}2}{40{\cdot}8}=29$ ins.

$$p_1=\frac{1{\cdot}32\times100}{29\times10}={\cdot}455$$

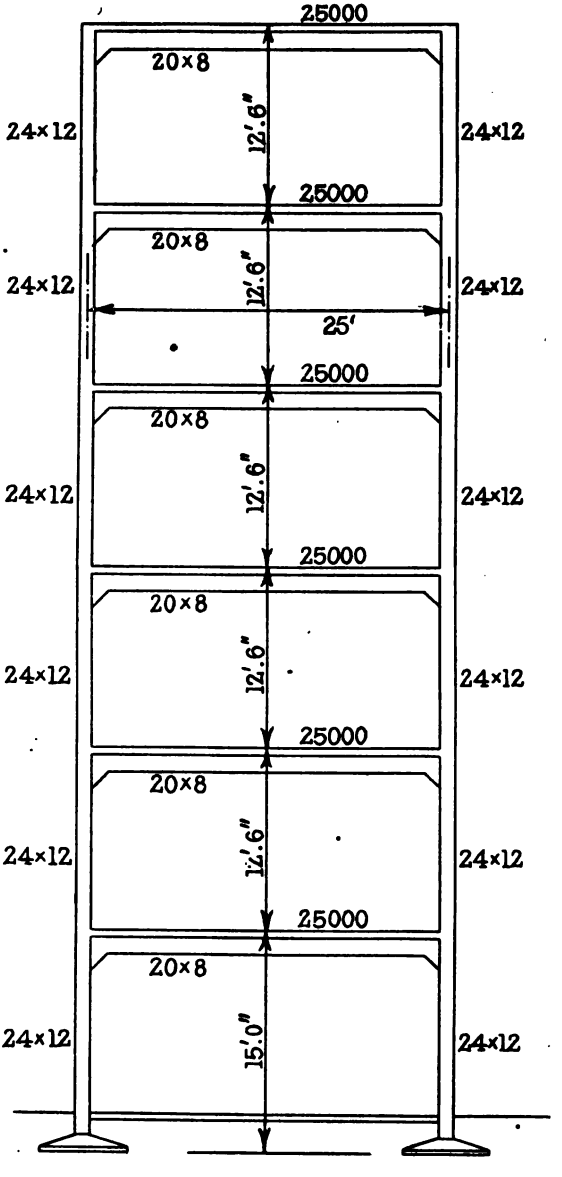



Fig. 68. Building. Beams of One Span Monolithic with Columns.

whence $t_1 = 28{\cdot}2$ (Fig. 32, Vol I.)

$a_1 = {\cdot}885$

$n_1 = {\cdot}34$ (Fig. 11, Vol. I.)

$n_1$ agrees well enough with the value assumed above (·36)

$$c = \frac{2P(e+f)}{nba} = \frac{25{,}000 \times 33''}{3{\cdot}4 \times 29 \times 8{\cdot}85}$$

$$= 945.$$

$$t = t_1 c = 28{\cdot}2 \times 945 = 26{,}700 \text{ lb./ins.}^2$$

It will be seen the more accurate calculation gives a stress (945) 40 % greater than the simple calculation (678).

The more accurate calculation is however admittedly too cumbersome for practice except in special cases, and having regard to the fact that many have ignored these eccentric stresses altogether, we may be satisfied with the simple method.

It is true that the factor of safety is then $\frac{40}{140} = 28\frac{1}{2}$ % less than that assumed, but on the other hand, a less factor is necessary when the unknown quantities are eliminated, of which this eccentric stress is the most important.

Note in passing that the reduction of stress effected by increasing the depth of column has been small—940 to 678·5 simple calculation, 1120 to 945 accurate calculation.

In the last calculation, we assumed the neutral axis to be on the centre line in calculating the moment of inertia, whereas actually it is 3·4″ from the compressed face. The error involved here is small as may be seen by calculating it now about the centre line as before, but taking the concrete as acting to the neutral axis only. This gives

Compression concrete $\frac{24 \times 6^3}{3} = 1730$

less concrete unstressed between centre line and neutral axis $\frac{24 \times 2{\cdot}6^3}{3} = \underline{141}$

1589

Compression steel (as before) 296

Tension steel (as before) 317

2202 ins.$^4$

which differs little from the previous value (2343).

In both the cases in which we have compared stresses by the simple and accurate method, the percentage of steel in the column was small (about 1 % of the total area). Where greater percentages are used, the difference is much reduced.

Coming now to the floor below, the $KC$ is now the sum of the $KC$'s of the columns above and below the joint.

Hence, in our case,

$$\frac{KC}{B} = 2 \times 4{\cdot}87 = 9{\cdot}74$$

whence $F_{14} = {\cdot}83$ (from Fig. 64)

and $$M = {\cdot}83 \times \frac{25{,}000 \times 300}{12}$$

$$= 520{,}000 \text{ in. lbs.}$$

This moment is taken, half on the column above, and half below.

Hence the stress in the column due to bending

$$= \frac{260{,}000 \times 6}{4170} = 375$$

$$\text{Direct stress} = \frac{2 \times 12{,}500}{325} = 77$$

$$\text{Total stress} = 452$$

At the other floors the calculation will be the same, the bending stress remaining the same, and the direct stress increasing by 38·5 lbs./ins.$^2$ at each floor.

When we come to first floor, there is a slight difference. Here the value of $K$ for the lower column is only 4 because it is fixed in direction at the bottom (see Vol. I, pp. 125 and 126). Also the height here is 15′ 0″.

Here $$KC \text{ above} = 6 \times \frac{4170}{150} = 167$$

$$KC \text{ below} = 4 \times \frac{4170}{180} = 93$$

$$\text{sum} = 260$$

$$\therefore \frac{KC}{B} = \frac{260 \times 300}{10{,}300} = 7{\cdot}55$$

whence $$F_{14} = {\cdot}79$$

$$M \text{ in col.} = {\cdot}79 \times \frac{25{,}000 \times 300}{12}$$

$$= 495{,}000 \text{ in. lbs.}$$

Of this $$\frac{167}{260} \times 495{,}000 = 318{,}000 \text{ in. lbs.}$$

applies to the section *above* first floor where the bending stress is therefore

$$\frac{318{,}000 \times 6''}{4170} = 458$$

and the direct stress

$$\frac{5 \times 12{,}500}{325} = 192\tfrac{1}{2}$$

$$\text{Total stress} = 650\tfrac{1}{2}$$

The moment immediately below the first floor is

$$\frac{93}{260} \times 495{,}000 = 177{,}000 \text{ in. lbs.}$$

and the bending stress therefore

$$\frac{177{,}000 \times 6}{4170} = 255$$

and the direct stress $\dfrac{6 \times 12{,}500}{325} = 231$

Total stress = 486

The following is therefore a summary of stresses:

| Position | Stresses | | |
|---|---|---|---|
| | Bending | Direct | Total |
| Below roof | 640 | 38½ | 678½ |
| Below 5th | 375 | 77 | 452 |
| „ 4th | 375 | 115½ | 490½ |
| „ 3rd | 375 | 154 | 529 |
| „ 2nd | 375 | 192½ | 567½ |
| Above 1st | 458 | 192½ | 650½ |
| Below 1st | 255 | 231 | 486 |

It will be seen that the greatest stress is under the roof, though the column is the same section all the way down. This will come as a shock to some, but it is undoubtedly often the case.

These stresses do *not* include those due to wind pressure, which would be important and greatest near the ground.

**Outside columns monolithic with Two or more Spans.** From Vol. I, p. 151, we have a formula (27) which gives the value for $Ea$ for the outer column of two spans.

Substituting this value in the general equation

$$M = KCEa$$

we have

$$Ea = -\frac{l}{12}\left[\frac{W_l\{(K_1C_1+4B)(K_2C_2+10B)-4B^2\}-W_d\{2B(K_1C_1+6B)\}}{(K_1C_1+4B)\{(K_1C_1+4B)(K_2C_2+8B)-8B^2\}}\right] \quad . \quad . \quad . \quad (1)$$

Let the student keep a stout heart. There is no need for him to use this in practice.

Now $K_1C_1$ refers to the outside columns and $K_2C_2$ to the interior ones.

In good practice the size of columns is largely determined by bending moments, and therefore though the load on outer columns is about one-half that on inside columns, the size of columns varies very little between the two.

Hence we may put $K_1C_1 = K_2C_2$ and the equation then reduces to

$$M = -\frac{l}{12}\left[\frac{W_t(D^2 + 14DB + 36B^2) - W_d(2BD + 12B^2)}{(D + 4B)(D^2 + 12BD + 24B^2)}\right] \times D \quad . \quad . \quad . \quad (2)$$

where $D = KC$ (for shortness)

$W_t$ = total load

$W_d$ = dead load.

It is convenient to write this as

$$M = -\frac{Wl}{12} \cdot F_{15}$$

where $W$ is the total load, and $F_{15}$ is a factor derived from the expression in the square brackets in (2) by taking $W_t$ outside and giving $\frac{W_d}{W_t}$ its value appropriate to any particular case, as below.

Thus, for all load live, $\frac{W_d}{W_t} = 0$

all load dead, $\frac{W_d}{W_t} = 1$

live load = dead, $\frac{W_d}{W_t} = \frac{1}{2}$

live load = 3 dead, $\frac{W_d}{W_t} = \frac{1}{4}$

The author has calculated the value of $F_{15}$ for these cases, and they lie so close that they cover all practical cases near enough. They are given in the following table.

These results have also been plotted on Fig. 64, values of $\frac{KC}{B}$ up to 30, and on an enlarged scale on Fig. 65 for values of $\frac{KC}{B}$ up to 1.

Comparing the results with $F_{14}$ (for one span) it will be seen

TABLE IV.

Values of $F_{15}$ for Moments in Outside Columns of Two Spans (or more).

| $\frac{KC}{B}$ | $\frac{B}{KC}$ | $F_{15}$ | | | |
|---|---|---|---|---|---|
| | | All load live | Live = 3 dead | Live = dead | All load dead |
| ·1 | 10 | ·036 | ·033 | ·031 | ·0246 |
| ·2 | 5 | ·069 | ·064 | ·059 | ·048 |
| ·5 | 2 | ·157 | ·145 | ·134 | ·111 |
| 1 | 1 | ·281 | ·255 | ·235 | ·195 |
| 2 | ·5 | ·435 | ·41 | ·385 | ·333 |
| 5 | ·2 | ·666 | ·647 | ·61 | ·555 |
| 10 | ·1 | ·805 | ·783 | ·76 | ·715 |
| 20 | ·05 | ·9 | ·885 | ·87 | ·833 |
| 30 | ·033 | ·937 | ·925 | ·914 | ·89 |
| ∞ | 0 | 1·0 | 1·0 | 1·0 | 1·0 |

that they vary but little, and the difference between the case of 2 spans and 3 is greatly less than between 1 span and 2.

$F_{15}$ may therefore be used for all cases of 2 spans or more.

With what has been said in regard to one span, their use and application need no further comment.

**Other arrangements of load.** For other arrangements of load, such as point loads at mid-span, the factors $F_{14}$ and $F_{15}$ apply rigidly as before, but are to be multiplied in all cases by $M_f$, where $M_f$ is the fixing moment at the fixed ends of a beam fixed at both ends and loaded in accordance with the case under consideration.

As was explained in Chapter X, $M_f$ has the following values:

| Loading | Values of $M_f$ |
|---|---|
| Uniform load ... ... ... | $-\frac{Wl}{12}$ |
| Point loads at mid-span ... | $-\frac{Wl}{8}$ |
| 2 point loads at third points ... | $-\frac{Wl}{9}$ |

# CHAPTER XII

## MOMENTS IN INTERIOR COLUMNS

*Case* 1. **Interior Column supporting a Continuous Beam of Two Spans, ends free. Uniform loading.**

From *Reinforced Concrete Design*, Vol. I, p. 131, we have a formula for $Ea$. Substituting this value in the general equation

$$M = KCEa^*$$

we have

$$M = -\frac{Wl}{8} \cdot \frac{KC}{KC + 6B}^* \quad . \quad . \quad . \quad . \quad (1)$$

where $W$ = live load uniformly distributed, $KC$ and $B$ have same meanings as before,

or

$$M = -\frac{Wl}{8} \cdot F_{13}$$

if $F_{13}$ has the value

$$\frac{KC}{KC + 6B}^*$$

This is evaluated in the following table and plotted on a curve in Fig. 69, and on an enlarged scale for values of $\frac{KC}{B}$ up to 1 on Fig. 69.

TABLE V.

| $\frac{KC}{B}$* | $F_{13}$ |
|---|---|
| 0 | 0 |
| ·1 | ·0164 |
| ·2 | ·0322 |
| ·5 | ·077 |
| 1·0 | ·143 |
| 2 | ·25 |
| 5 | ·455 |
| 10 | ·625 |
| 20 | ·77 |
| 30 | ·833 |
| ∞ | 1·0 |

* For meaning of symbols see p. 124.

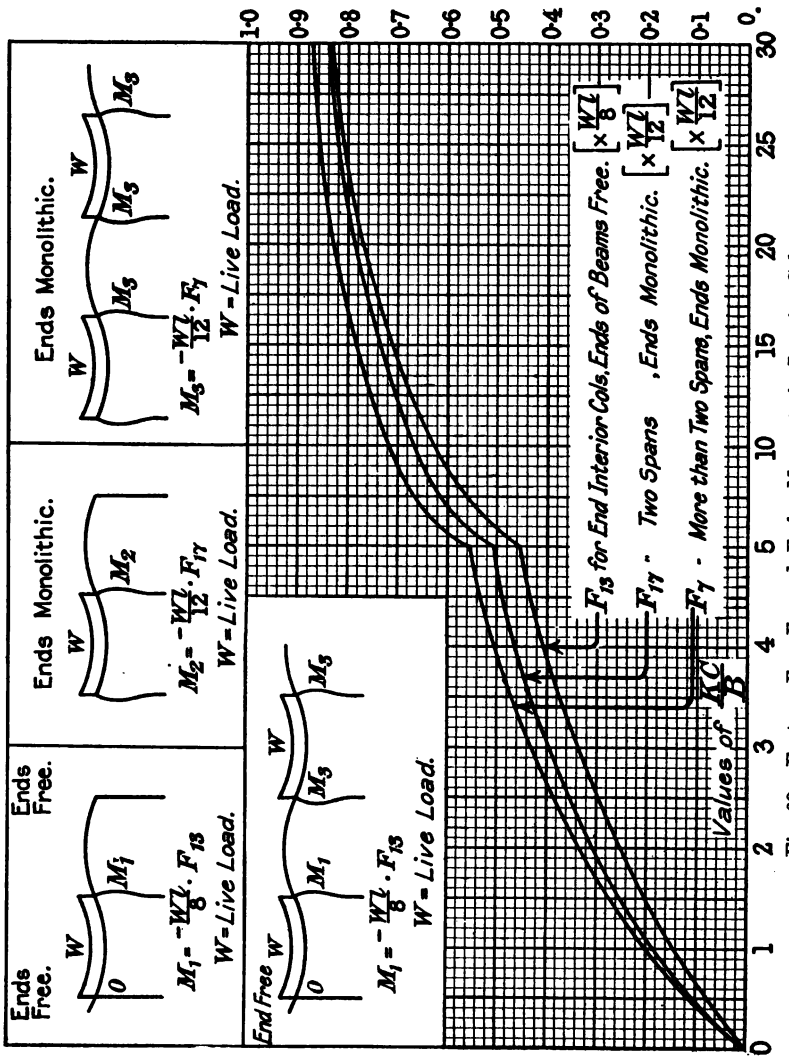



Fig. 69. Factors $F_{13}$, $F_{17}$ and $F_7$ for Moments in Interior Columns.

Notice particularly that this has to be multiplied by $\frac{Wl}{8}$, not $\frac{Wl}{12}$ as do some of the factors, and that $W$ is the live load only. The application of this is quite simple if the preceding chapter was followed.

Note, however, that the moment only exists with one span uniformly loaded and the other unloaded. The direct reaction on the centre column is then

$$\tfrac{5}{16}W_l + \tfrac{5}{8}W_d$$

which may be considerably less than when both spans are loaded, when

$$R = \tfrac{5}{8}W_l + \tfrac{5}{8}W_d$$

This applies to the load due to the unequally loaded floor only, and clearly the upper floors, if any, may be fully loaded.

For interior columns, the value of $K$ may in all cases be taken as 4 (which assumes the column retains its original slope at the floor above and below).

*Other distributions of load.*

For other load distributions, the Factor $F_{13}$ is unchanged, but should be multiplied by the fixing moment at the fixed end of a beam fixed one end and freely supported at the other, and loaded as the case under consideration.

As explained in Chapter X, this moment has the values

Uniform distribution $\frac{Wl}{8}$

Point load mid-span $\frac{3}{16}Wl$

Two point loads at third points $\frac{1}{6}Wl$

$W$ is live load only.

*Case* 2. **Interior Column supporting a Continuous Beam of Two Spans, ends monolithic.**

From Vol. I, p. 276, is given an expression for $Ea_2$, the slope at this column. Substituting this value in the general

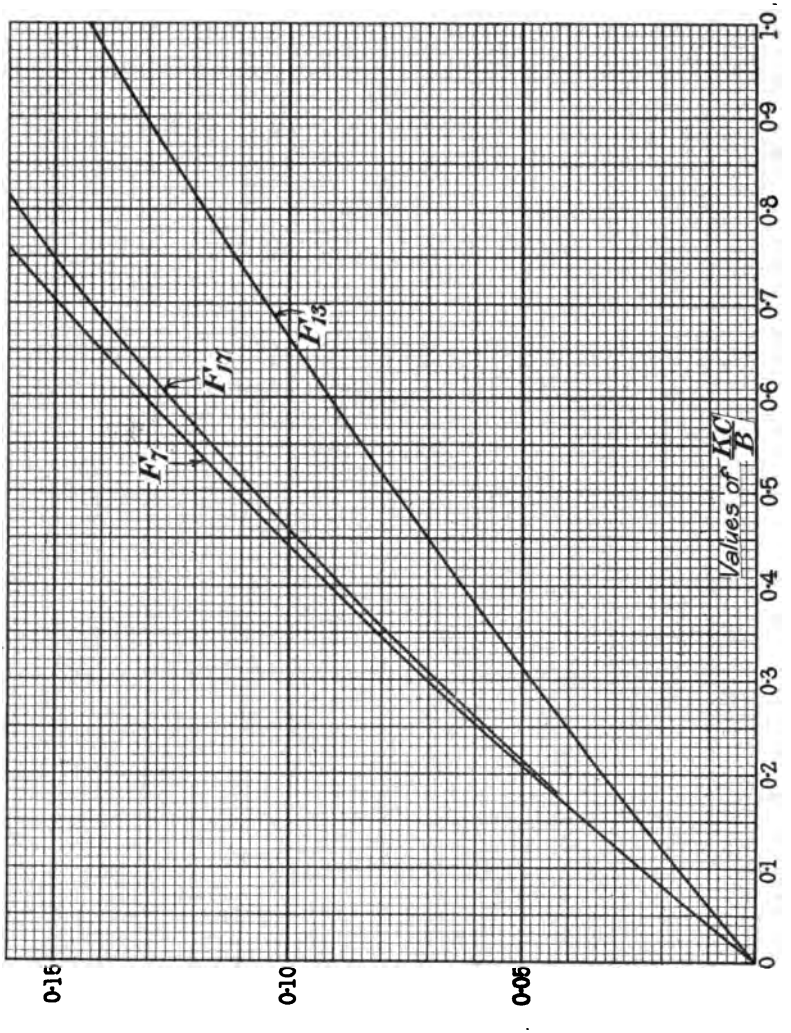



Fig. 70. Factors $F_{13}$, $F_{17}$ and $F_7$ for Moments in Interior Columns. Values $\frac{KC}{B}$ up to 1.

equation $M = KCE\alpha$, and putting the $KC$'s for the outer and inner columns equal, we have

$$M = -\frac{Wl}{12}\frac{(KC + 6B)\,KC}{(KC + 4B)\,(KC + 8B) - 8B^2} \quad . \quad (2)$$

or $$M = -\frac{Wl}{12} \cdot F_{17}$$

where $$F_{17} = \frac{(KC + 6B)\,KC}{(KC + 4B)\,(KC + 8B) - 8B^2}$$

Values of $F_{17}$ are plotted on Figs. 69 and 70 and given in the following table.

TABLE VI.

| $\frac{KC}{B}$ | $F_{17}$ |
|---|---|
| 0 | 0 |
| ·1 | ·0241 |
| ·2 | ·0469 |
| ·5 | ·1075 |
| 1·0 | ·189 |
| 2 | ·307 |
| 5 | ·505 |
| 10 | ·666 |
| 20 | ·783 |
| 30 | ·84 |
| ∞ | 1·0 |

*Note.* $K = 4$ as before; $W$ = live load as before; moment only exists with one bay only loaded as before.

The reaction is then

$$R = \tfrac{5}{8}W_l + \tfrac{5}{4}W_d \text{ for small values of } \frac{KC}{B}$$

but is $$R = \frac{W_l}{2} + W_d \text{ for very high values of } \frac{KC}{B}$$

*Other distributions of load.* For other load distributions, the factor $F_{17}$ remains, but it is to be multiplied by the fixing moment at the end of a beam fixed both ends and loaded as the case under consideration, for which the values are

Uniform distribution $-\frac{Wl}{12}$

Point load at mid-span $-\frac{Wl}{8}$

Two point loads at third points $-\frac{Wl}{9}$

*Case* 3. **End but one Columns, More than Two Spans, Ends of Beams freely Supported.**

This case gives the same moments as in Case 1, and $F_{13}$ may be used and multiplied by $\frac{Wl}{8}$ exactly as was done in Case 1.

The reaction with the end span loaded as shown and the adjacent one unloaded will be $\frac{5}{8}W_l + W_d$ (near enough) for small values of $\frac{KC}{B}$.

And $\frac{W_l}{2} + W_d$ for very large values of $\frac{KC}{B}$.

The remarks about other load distributions in Case 1 apply here also.

*Case* 4. **End but one Columns, More than Two Spans, Ends of Beam Monolithic, and all other Interior Columns.**

It was shown in Vol. I, p. 284, that for interior columns far removed from the ends of a continuous beam, the worst moments occur with alternate spans loaded and the remainder unloaded, and are then given by an expression for $Ea$, which substituted in the general equation $M = KCEa$ gives

$$M = -\frac{Wl}{12} \cdot \frac{KC}{KC + 4B}$$

or

$$= -\frac{Wl}{12} \cdot F_7$$

where

$$F_7 = \frac{KC}{KC + 4B}$$

Values of $F_7$ are given in the following table, and are plotted in Figs. 69 and 70.

It may be shown that this factor applies also quite closely to the case of the end but one column of a series of more than two spans when the ends of the beam are monolithic with the columns.

The application is obvious from what was said in this and the preceding chapter.

TABLE VII.

| $\frac{KC}{B}$ | $F_7$ |
|---|---|
| 0 | 0 |
| ·2 | ·0477 |
| ·4 | ·091 |
| ·6 | ·1305 |
| ·8 | ·1667 |
| 1·0 | ·2 |
| 1·25 | ·238 |
| 1·67 | ·295 |
| 2·5 | ·385 |
| 5 | ·555 |
| 10 | ·714 |
| 20 | ·830 |
| 30 | ·870 |
| ∞ | 1·0 |

*Note.* $W$ = live load; moment exists with alternate bays unloaded only, when the reactions are

$$R = \frac{W_l}{2} + W_d$$

For other distributions of load, the reader is referred to Case 2.

# PART III

## CHAPTER XIII

### STANDARD COLUMNS AND BEAMS

Every designer experienced in steel construction knows what tremendous assistance and saving in time has resulted from the existence of standard sizes of joists.

The properties of standard sections can be worked out fully in great detail and with great accuracy once and for all, and be available for ever after by mere inspection.

The systematic calculation of bending moments in columns, which has so long been shirked at great risk—which was generally not realised—becomes so simple when the curves in Part II are combined with the use of standard sections that from henceforth a man will be more knave than fool who omits to design for them.

Table I gives the properties of a set of standard square columns varying from 6″ × 6″ to 24″ × 24″ with intervals of 2″ in the side. The reinforcement is something just over 1 per cent., measured on the whole area of the column so far as is consistent with the sizes of commercial sections of rods.

In the case of the 6″ × 6″ up to and including the 16″ × 16″, there are four rods per column, one in each corner. From the 18″ × 18″ up to 24″ × 24″ there are eight rods in each column situated one in each corner and one in the middle of each face. The size of the rods varies from $\frac{1}{2}$″ to 1″ and, in the author's opinion, it is undesirable except in exceptional cases to use larger rods in the columns since they are very frequently unable to develop their full strength in the necessary length having regard to their adhesion properties.

In column 4 the area of the concrete is calculated taking the whole section of the column. Column 5 is the area of the steel which in column 6 is multiplied by 14, (15 − 1), so as to give the equivalent area of concrete allowing for the concrete which the steel has displaced, and columns 4 and 6 are added

## TABLE I. Properties of Standard Square Columns.

[NOT calculated in accordance with London County Council Regulations.]

| Size of column | Vertical rods | | Areas | | | | Area steel in two faces | Distance steel from neutral axis | Moments of Inertia* | | | | Radius of gyration |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | No. | Diam. | Concrete | Steel | Steel express'd as equiv. concrete | Total area concrete units | | | *I* steel Steel units | *I* steel Equiv. concrete units | *I* concrete | Total *I* Concrete units | |
| ins. | | ins. | ins.$^2$ | ins.$^2$ | ins.$^2$ | ins.$^2$ | ins.$^2$ | ins. | ins.$^4$ | ins.$^4$ | ins.$^4$ | ins.$^4$ | ins. |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
| 6×6 | 4 | ½ | 36 | ·8 | 11·2 | 47·2 | ·8 | 1 | ·8 | 11·2 | 108 | 119·2 | 1·58 |
| 8×8 | 4 | ½ | 64 | ·8 | 11·2 | 75·2 | ·8 | 2 | 3·2 | 45 | 340 | 385 | 2·26 |
| 10×10 | 4 | ⅝ | 100 | 1·2 | 16·8 | 116·8 | 1·2 | 3 | 10·8 | 152 | 840 | 992 | 2·91 |
| 12×12 | 4 | ⅞ | 144 | 1·76 | 24·6 | 168·6 | 1·76 | 4 | 27·9 | 395 | 1740 | 2135 | 3·56 |
| 14×14 | 4 | ⅞ | 196 | 2·4 | 33·6 | 230 | 2·4 | 5 | 60 | 840 | 3100 | 3940 | 4·14 |
| 16×16 | 4 | 1 | 256 | 3·14 | 44 | 300 | 3·14 | 6 | 114 | 1590 | 5500 | 7090 | 4·86 |
| 18×18 | 8 | ¾ | 324 | 3·52 | 49 | 373 | 2·64 | 7 | 129 | 1810 | 8700 | 10510 | 5·3 |
| 20×20 | 8 | ⅞ | 400 | 4·8 | 67 | 467 | 3·6 | 8 | 231 | 3240 | 13300 | 16540 | 5·94 |
| 22×22 | 8 | 1 | 484 | 6·28 | 88 | 572 | 4·7 | 9 | 381 | 5320 | 19600 | 24920 | 6·59 |
| 24×24 | 8 | 1 | 576 | 6·28 | 88 | 664 | 4·7 | 10 | 470 | 6600 | 27400 | 34000 | 7·14 |
| | | | whole area measured | | modular ratio 15 | | | steel taken 2″ in | | modular ratio 15 | whole area measured | | |

* See footnote p. 149.

to give 7, the total equivalent area of the column in terms of concrete units.

Coming now to the moments of inertia*, in the case of the columns reinforced with four bars, all four bars lie at the same distance from the neutral axis. The moment of inertia of the steel is therefore obtained by multiplying the area of the rods by the square on their distance from the neutral axis. This is done in columns 8–10 and in column 11 the moment of inertia of the steel is multiplied by 14 (15 – 1) so as to convert it to the equivalent in terms of concrete units.

In determining the distance of the steel from the neutral axis in column 9, this has in all cases been taken so that the centre line of the bar is 2″ in from the outer face. This gives exactly $1\frac{1}{2}$″ cover as required by the L.C.C. Regulations where the bars are 1″ diameter and slightly more for smaller bars. Thus for a $\frac{1}{2}$″ bar it gives $1\frac{3}{4}$″ cover. It is appreciated that in some cases the columns might be constructed with slightly smaller cover to the steel but it is difficult to be quite sure of the exact position of the steel and in any case it is rather a good thing to slightly penalise the more slender columns where a slight flaw in the workmanship or a small chip of wood being left in is much more dangerous than it is in the case of larger columns, where also inspection is easier and more likely to be efficient.

In any case it will be seen that the strength given in Tables I and II is so much greater than what is allowed by the L.C.C. Regulations that no one will object to them on the score that the design is uneconomical.

In columns 18″–24″, in which there are eight bars instead of four, it is clear that two of these bars lie on the neutral axis and therefore add nothing to the moment of inertia. These bars are therefore neglected in calculating the moment of inertia of the steel as will be seen by inspection of column 8, comparing it with column 5.

Column 12 is the moment of inertia of the concrete $\left(\frac{BD^3}{12}\right)$ and column 13 the total equivalent moment of inertia of the whole column in concrete units, by adding columns 11 and 12.

* The moments of inertia only apply with the column in compression on the whole area. When much tension occurs about half the values given apply. See calculation on pp. 127 and 131.

## TABLE II. Properties of 24″ Rectangular Columns.

[NOT calculated in accordance with London County Council Regulations.]

| Size of col. | Vertical rods | | | Areas | | | | Area steel in two long faces | Distance of steel from N.A. | Moments of Inertia * | | | | Radius of gyration |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | No. | Diam. | Arrangement | Concrete | Steel | Steel as equiv. concrete units | Total area concrete units | | | *I* steel Steel units | *I* steel Equiv. concrete units | *I* concrete | Total *I* Concrete units | |
| ins. | | ins. | | ins.² | ins.² | ins.² | ins.² | ins.² | ins. | ins.⁴ | ins.⁴ | ins.⁴ | ins.⁴ | ins. |
| 1 | 2 | 3 | 3a | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
| 24×6 | 8 | ¾ | 4 each long face | 144 | 1·6 | 22·4 | 166·4 | 1·6 | 1 | 1·6 | 22·5 | 432 | 454·5 | 1·65 |
| 24×8 | 8 | ¾ | 4 ,, ,, | 192 | 1·6 | 22·4 | 214 | 1·6 | 2 | 6·4 | 90 | 1020 | 1110 | 2·28 |
| 24×10 | 8 | ¾ | 4 ,, ,, | 240 | 2·4 | 33·6 | 274 | 2·4 | 3 | 21·6 | 302 | 2016 | 2318 | 2·91 |
| 24×12 | 6 | ¾ | 3 ,, ,, | 288 | 2·64 | 37 | 325 | 2·64 | 4 | 42·2 | 590 | 3580 | 4170 | 3·58 |
| 24×14 | 6 | ¾ | 3 ,, ,, | 336 | 3·6 | 50·4 | 386 | 3·6 | 5 | 90 | 1260 | 5300 | 6560 | 4·12 |
| 24×16 | 6 | 1 | 3 ,, ,, | 384 | 4·7 | 66 | 450 | 4·7 | 6 | 170 | 2380 | 8250 | 10630 | 4·86 |
| 24×18 | 8 | 1 | 3 each face | 432 | 6·28 | 88 | 520 | 4·7 | 7 | 230 | 3220 | 11600 | 14820 | 5·33 |
| 24×20 | 8 | 1 | 3 ,, | 480 | ,, | ,, | 568 | 4·7 | 8 | 300 | 4200 | 16000 | 20200 | 5·95 |
| 24×22 | 8 | 1 | 3 ,, | 528 | ,, | ,, | 616 | 4·7 | 9 | 382 | 5320 | 21400 | 26720 | 6·57 |
| 24×24 | 8 | 1 | 3 ,, | 576 | ,, | ,, | 664 | 4·7 | 10 | 472 | 6600 | 27400 | 34000 | 7·14 |
| | | | | whole area measured | centre of bars 2″ in | modular ratio 15 | | | bars 2″ in | | modular ratio 15 | *I* measured about longer axis | | |

* See footnote p. 149.

The radius of gyration, in column 14, sometimes required in connection with buckling, is obtained by calculating $\sqrt{\frac{I}{A}}$, taking $I$ and $A$ from columns 7 and 13 respectively. The author still thinks the treatment of columns given in *Reinforced Concrete Design*, Vol. I, p. 113, more practical and scientific in the real sense of agreeing well with experimental evidence.

Now, in the case of columns in walls it is frequently desirable to adopt some definite width for such columns which will be retained on successive story heights, any variation in size required being obtained by varying the depth. It is clear that this will generally give a more pleasing architectural effect from the outside and also enables windows on the different floors to be the same width which is also a great advantage from the point of view of standardisation of windows. It is therefore convenient to have in addition to the square columns similar "standard rectangular columns," and a set of such standards is given in Table II of the properties of 24″ rectangular columns.

The explanations that have just been given apply equally to this table except as follows. In the 24″ × 6″ column there are eight ½″ rods, four along each face. It is desirable to distribute the steel fairly uniformly over the concrete which is done more effectively by this design than if the steel were concentrated in the four corners with such a long rectangle. Such a column will, of course, require intermediate links in addition to those going all round the column since otherwise the intermediate rods would not be prevented from buckling sideways. As before, it is also considered an advantage to employ small rods and many of them, as their properties of adhesion are so much greater.

In regard to the moments of inertia, these are calculated about the longer axis of the column, i.e. the bending is assumed to take place in the direction of the short axis in the same way as the column would buckle and as it would bend in the case of wall columns with the long face along the wall and the main beams running into them from the inside of the building.

It will be clear from these tables that the moment of inertia or the equivalent area for any column can be read off at a glance and the saving in time which results in practice is tremendous.

## TABLE III. Properties of Standard Square Columns.

[Calculated to comply with London County Council Regulations.]

| Size of col. | Vertical rods | | Areas | | | | Area of steel in two faces | Distance of steel from N.A. | Moments of Inertia * | | | | Radius of gyration |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | No. | Diam. | Concrete inside core | Steel | Steel as equiv. concrete | Total area concrete units | | | *I* steel Steel units | *I* steel Concrete units | *I* concrete Inside core | Total *I* Concrete units | |
| ins. | | ins. | ins.² | ins.² | ins.² | ins.² | ins.² | ins. | ins.⁴ | ins.⁴ | ins.⁴ | ins.⁴ | ins. |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
| 6×6 | 4 | ½ | 9 | ·8 | 11·2 | 20·2 | ·8 | 1 | ·8 | 11·2 | 6·75 | 17·95 | ·94 |
| 8×8 | 4 | ⅝ | 25 | ·8 | 11·2 | 36·2 | ·8 | 2 | 3·2 | 45 | 52 | 97 | 1·64 |
| 10×10 | 4 | ¾ | 49 | 1·2 | 16·8 | 65·8 | 1·2 | 3 | 10·8 | 152 | 201 | 353 | 2·32 |
| 12×12 | 4 | ⅞ | 81 | 1·76 | 24·8 | 105·8 | 1·76 | 4 | 27·9 | 395 | 547 | 942 | 2·98 |
| 14×14 | 4 | 1 | 121 | 2·4 | 33·6 | 154·6 | 2·4 | 5 | 60 | 840 | 1230 | 2070 | 3·66 |
| 16×16 | 4 | 1 | 169 | 3·14 | 44 | 213 | 3·14 | 6 | 114 | 1590 | 2390 | 3980 | 4·32 |
| 18×18 | 8 | ¾ | 225 | 3·52 | 49 | 274 | 2·64 | 7 | 129 | 1810 | 4220 | 6030 | 4·7 |
| 20×20 | 8 | ⅞ | 289 | 4·8 | 67 | 356 | 3·6 | 8 | 231 | 3240 | 6970 | 10210 | 5·35 |
| 22×22 | 8 | 1 | 361 | 6·28 | 88 | 449 | 4·7 | 9 | 381 | 5320 | 10900 | 16220 | 6·01 |
| 24×24 | 8 | 1 | 441 | 6·28 | 88 | 529 | 4·7 | 10 | 470 | 6600 | 16250 | 22850 | 6·56 |
| | | | allowing 1½″ cover | | modular ratio 15 | | | steel taken as 2″ in | | modular ratio 15 | neglecting outer 1½″ concrete | | neglecting outer 1½″ concrete |

* See footnote p. 149.

These two Tables, I and II, are not in accordance with the L.C.C. Regulations because in these regulations it is stipulated that the area of concrete to be taken into account in calculating the column is to be confined to what is called the "hooped core," this being that portion of the column lying inside the lateral binding, i.e. the portion inside the line joining the outer faces of the vertical bars. It is further stipulated that the cover of concrete in the columns is not to be less than $1\frac{1}{2}''$ or the diameter of the bar, and as we are not using bars larger than $1''$ this means $1\frac{1}{2}''$. A further stipulation is that no rods are to be less than $\frac{1}{2}''$ diameter.

Coming now to Table III, which is calculated in accordance with the L.C.C. Regulations as far as a human being can understand them, we have in column 4 of the $6'' \times 6''$ column to consider not $36''$ square of concrete but only $9''$, since, when we have deducted the $1\frac{1}{2}''$ cover all round the $6'' \times 6''$ column, only $3''$ square is left, and similarly for all other columns $3''$ is deducted from sizes in calculating the area of concrete in the hooped core.

The area of steel and the equivalent area are calculated as before and no further comment is necessary.

Coming now to the calculation of the moments of inertia, the same thing applies. We have to consider the moment of inertia, not of the whole area of the column, but only of that portion after the outside cover has been neglected, which, of course, very greatly reduces it. With these remarks the calculations will be quite clear.

The same applies to Table IV in which the rectangular columns are calculated in accordance with the L.C.C. Regulations. Here the $24'' \times 6''$ column has to be calculated as if it were $21'' \times 3''$, and while the area is thus reduced from 166·4 to 85·4, roughly speaking 50 per cent., the moment of inertia is reduced from 454·5 to 69·8 which is, of course, a very drastic alteration. Naturally for larger columns the effect is not so marked and for the $24'' \times 24''$ column the area is reduced from 664 to 529 and the moment of inertia from 34,000 to 22,850. The object of the L.C.C. Regulations in this respect is of course to give some sort of protection against fire so that even if the outside $1\frac{1}{2}''$ of concrete were damaged by intense heat the

## TABLE IV. Properties of 24″ Rectangular Columns.

[Calculated in accordance with London County Council Regulations.]

| Size of col. | Vertical rods | | | Areas | | | | Area steel in two long faces | Distance of steel from N.A. | Moments of Inertia * | | | | Radius of gyration |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | No. | Diam. | Arrangement | Concrete | Steel | Steel as equiv. concrete units | Total area concrete units | | | *I* steel Steel units | *I* steel Equiv. concrete units | *I* concrete | Total *I* Concrete units | |
| ins. | | ins. | | ins.² | ins.² | ins.² | ins.² | ins.² | ins. | ins.⁴ | ins.⁴ | ins.⁴ | ins.⁴ | ins. |
| 1 | 2 | 3 | 3*a* | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
| 24× 6 | 8 | ½ | 5 each long face | 63 | 1·6 | 22·4 | 85·4 | 1·6 | 1 | 1·6 | 22·5 | 47·3 | 69·8 | ·905 |
| 24× 8 | 8 | ½ | 4 ,, ,, | 105 | 1·6 | 22·4 | 127·4 | 1·6 | 2 | 6·4 | 90 | 219 | 309 | 1·56 |
| 24×10 | 8 | ½ | 4 ,, ,, | 147 | 2·4 | 33·6 | 180·6 | 2·4 | 3 | 21·6 | 302 | 600 | 902 | 2·23 |
| 24×12 | 6 | ½ | 3 ,, ,, | 189 | 2·64 | 37 | 226 | 2·64 | 4 | 42·2 | 590 | 1275 | 1865 | 2·87 |
| 24×14 | 6 | ½ | 3 ,, ,, | 231 | 3·6 | 50·4 | 281·4 | 3·6 | 5 | 90 | 1260 | 2330 | 3590 | 3·57 |
| 24×16 | 6 | 1 | 3 ,, ,, | 273 | 4·7 | 66 | 339 | 4·7 | 6 | 170 | 2380 | 3850 | 6230 | 4·29 |
| 24×18 | 8 | 1 | 3 each face | 315 | 6·28 | 88 | 403 | 4·7 | 7 | 230 | 3220 | 5910 | 9130 | 4·75 |
| 24×20 | 8 | 1 | 3 ,, | 357 | ,, | ,, | 445 | 4·7 | 8 | 300 | 4200 | 8600 | 12800 | 5·35 |
| 24×22 | 8 | 1 | 3 ,, | 399 | ,, | ,, | 487 | 4·7 | 9 | 382 | 5320 | 12000 | 17320 | 5·95 |
| 24×24 | 8 | 1 | 3 ,, | 441 | ,, | ,, | 529 | 4·7 | 10 | 472 | 6600 | 16250 | 22850 | 6·56 |
| | | | | neglecting outer 1½″ concrete | | modular ratio 15 | | | steel taken as 2″ in | *I* meas. about longer axis | neglecting outer 1½″ concrete | neglecting outer 1½″ | | |

* See footnote p. 149.

inside should still be of sufficient strength to carry the building without any reduction in the factor of safety. No doubt for buildings in the City of London this is a wise precaution to take, and for such purposes Tables III and IV may then be used, but there will, of course, be many cases, as, for example, in the case of a viaduct crossing the open country, where such fire precautions are quite unnecessary and for such cases the much greater economy of Tables I and II would appear to be justified.

It should be pointed out that in compiling Tables III and IV the centre line of the steel bars is also taken at 2″ in from the face of the column which agrees exactly with the L.C.C. Regulations in the case of 1″ bars and gives $\frac{1}{4}$″ more cover than is called for in the case of those with $\frac{1}{2}$″ bars.

Coming now to standard beams, a series of standard T beams is given in Table V which will be found to be of immense value in practice.

The beams adopted vary from 6″ wide and 12″ overall depth to 12″ wide and 30″ overall depth. The various columns in the table are self-explanatory, but the following remarks may be helpful. The radius arm in column 7 is obtained by multiplying the effective depth by ·88, in other words taking the neutral axis as a line ·36 of the effective depth from the top, which condition obtains when stresses of 600 exist in the concrete and 16,000 in the steel or any other stresses having the same ratio.

The section modulus in steel units given in column 8 is obtained by multiplying the area of steel in column 6 by the radius arm in column 7.

In column 9 is given the width of slab which is necessary for the section modulus to apply and in the calculation of this figure it is assumed that the slab is of the thickness equal to the depth of the neutral axis, namely, ·36 times the effective depth, which is given in column 10, the effective depth being given in column 11. The necessary width in column 9 is calculated by finding the width necessary to give a percentage of steel ·675 per cent., measured on this width multiplied by the effective depth *d*. If in any practical example the thickness of the slab at the top of the beam is less than that given in column 10, then a proportionately greater width of slab is needed to act as a

## TABLE V. Standard T Beams.

[Calculated in accordance with London County Council Regulations and with *Reinforced Concrete Design.* Vol. I.]

| Name | Overall depth | Width | Tension steel | | | Radius arm | Section modulus | Width of slab needed | Thickness of slab needed $n$ | Effective depth $d$ | Moment of Inertia | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | Bottom layer | Top layer | Total area | | Steel Units | | | | Steel units | Concrete units |
| | ins. | ins. | | | ins.[2] | ins. | ins.[3] | ins. | ins. | ins. | ins.[4] | ins.[4] |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
| 12×6 | 12 | 6 | 2—$\frac{1}{2}$″ | 2—$\frac{1}{2}$″ | 1·2 | 9·5 | 11·4 | 10·7 | 3·88 | 10$\frac{1}{4}$ | 78·8 | 1180 |
| 14×6 | 14 | 6 | 2—$\frac{5}{8}$″ | 2—$\frac{5}{8}$″ | 1·48 | 11·1 | 16·4 | 14·9 | 4·55 | 12$\frac{3}{8}$ | 133 | 2000 |
| 16×6 | 16 | 6 | 2—$\frac{3}{4}$″ | 2—$\frac{3}{4}$″ | 1·76 | 12·8 | 22·5 | 18 | 5·22 | 14$\frac{1}{2}$ | 219 | 3290 |
| 18×8 | 18 | 8 | 3—$\frac{5}{8}$″ | 3—$\frac{5}{8}$″ | 2·64 | 14·5 | 38·2 | 23·7 | 5·95 | 16$\frac{1}{2}$ | 405 | 6090 |
| 20×8 | 20 | 8 | 3—$\frac{3}{4}$″ | 3—1″ | 3·67 | 16·0 | 58·6 | 29·5 | 6·57 | 18$\frac{1}{4}$ | 687 | 10300 |
| 22×8 | 22 | 8 | 3—1″ | 3—1″ | 4·7 | 17·6 | 83 | 34·9 | 7·2 | 20 | 1065 | 16000 |
| 24×10 | 24 | 10 | 4—1″ | 4—1″ | 6·28 | 19·3 | 121 | 42·3 | 7·9 | 22 | 1710 | 25600 |
| 26×10 | 26 | 10 | 4—1″ | 4—1$\frac{1}{4}$″ | 8·02 | 20·9 | 168 | 50 | 8·5 | 23$\frac{3}{4}$ | 2550 | 38300 |
| 28×10 | 28 | 10 | 4—1$\frac{1}{4}$″ | 4—1$\frac{1}{4}$″ | 9·76 | 22·4 | 219 | 56·6 | 9·2 | 25$\frac{1}{2}$ | 3570 | 53600 |
| 30×12 | 30 | 12 | 5—1$\frac{1}{4}$″ | 5—1$\frac{1}{4}$″ | 12·2 | 24·2 | 295 | 66 | 9·9 | 27$\frac{1}{2}$ | 5200 | 78000 |

[2] Including slab thickness

[3] If slab is of less thickness than given in column 10, the width needed is greater than in column 9. If this cannot be provided, compression steel must be used.

compression boom than the figure given in column 9, and if it is found that this cannot be provided by the actual design then top reinforcement must be provided and designed to make the strength of the compression flange equal to that of the tension flange. In the calculation of the moment of inertia in column 12 in steel units we have multiplied the section modulus by the distance from the neutral axis to the centre of the steel. Since the depth of the neutral axis is $\cdot 36d$ the distance from the neutral axis to the steel is clearly $\cdot 64d$ and therefore column 12 is obtained by multiplying the section modulus in column 8 by $\cdot 64d$. This gives results in steel units and in 13 we give the result in concrete units by multiplying by the modular ratio 15 so as to have the moment of inertia of the beam expressed in the same units as for the columns, which is very convenient when calculating our ratio $\frac{KC}{B}$ which is frequently required, as throughout Part II.

It is thought that the use of this beam table will be obvious without further comment, but a few remarks may be helpful. Suppose the bending moment in a beam near mid-span is determined, we divide by the steel stress to be adopted and the result gives us the section modulus required, from which we select the beam from column 8, and all the other properties of the beam may then be read off by simple inspection.

It should be clearly understood that this table only applies to the central portions of the beam where it resists positive bending moments and acts as a T beam and that near the support where it acts as a rectangular beam the table must, of course, not be used. Here the very simple treatment given in Vol. I, Chap. II, will enable the sizes at this point to be determined readily.

It is believed that the properties in Table V comply with the Regulations of the L.C.C. for T beams.

It should be pointed out that these tables are very useful even when in some particular case a reason can be made out for departing from them. Thus, referring to Table V, supposing a $30'' \times 12''$ beam were required to be increased in strength by 20 per cent. without increasing the depth, all that is required is to put twelve $1\frac{1}{4}''$ bars instead of 10 and the area of steel and

the section modulus would then simply have to be increased by 20 per cent., which is much quicker than working them out afresh. Similarly the width of slab needed in the compression flange needs to be increased by 20 per cent. and the moment of inertia would then also be 20 per cent. greater. Thus the tables are quite useful even when slight variations are called for in any special cases. The same applies to the column sections. Thus, suppose we want the properties of a column 20″ × 10″ which is not given, this may be obtained at once from Table I by doubling the area of the 10″ × 10″ column. We then get a column of 20″ × 10″ with eight $\frac{5}{8}$″ rods and the equivalent area would be 116·8 × 2. The moment of inertia about the long axis, i.e. in the more flexible direction, would then clearly also be 992 × 2.

Tables III and IV of the properties calculated in accordance with the L.C.C. Regulations cannot, however, be compounded so readily because of the deduction of the cover of concrete. The best way to treat them then is by remembering that an L.C.C. 20″ × 10″ column with eight $\frac{5}{8}$″ rods will have the area of steel doubled and the area of effective concrete increased from 7 to 17, and the figures from columns 6 and 4 would therefore need to be increased in these ratios respectively and added together to give the new figure for the equivalent area. Similarly, column 12 for the moment of inertia would need increasing in the ratio of $\frac{17}{7}$, and column 11 would need doubling and the two figures thus obtained added together would give the new figure for column 13 of the total moment of inertia.

In the same way even if it is desired to alter the sizes of the bars the tables are useful. Thus, supposing in Table I for some special reason we were wanting to use a 16″ × 16″ column with eight 1″ bars instead of four. Clearly the new equivalent area would be 256 as before + 2 × 44 = 344, and the new moment of inertia would be

$$5500 \text{ (as before)} + 2 \times 1590 = 8680.$$

# CHAPTER XIV

## LIVE LOAD ALLOWANCE FOR ROLLING LOADS

With trains on viaducts, it is quite necessary to allow something for shock, or vibration. This allowance is often called "live load allowance," but live load is meant in quite a different sense from that in which we used it in Chap. I.

It is usual to allow 100 per cent. on the actual weight of axles for this vibration for main line traffic with reduction on long spans.

After a very careful study of the problem, and examining and designing many concrete bridges, the author finds the following formula, which he has devised, meets the case for high speed traffic.

If $W$ = actual loads to be carried, design for

$$W\left(1 + \frac{100}{100 + l}\right) = W\left(\frac{l + 200}{l + 100}\right) \quad . \quad . \quad . \quad (1)$$

to make allowance for impact at 60 miles an hour and for various spans.

In this formula $l$ is the span in feet.

This allows 100 per cent. for vibration on short spans, 50 per cent. on spans of 100 ft. and 25 per cent. on spans of 300 ft.

It is clear that on long spans the train takes quite an appreciable time to reach the centre even at high speeds, and therefore the main girders have their load applied gradually. Any cross girders or slabs would of course still be designed for the 100 per cent. allowance.

In the case of a train travelling at 60 miles an hour or 88 ft. a second, it takes two seconds for the loco to reach the middle of a bridge 352 ft. long, and nearly four seconds before the whole bridge is covered.

It is usual with steel bridges to write formula (1) so that the allowance is not reduced materially till greater spans are reached.

Thus the American Bridge Co.'s formula

$$W\left(1+\frac{300}{300+l}\right) \text{ or } \left(\frac{l+600}{l+300}\right)W$$

is often used. It must be remembered that concrete bridges are heavier than steel, and therefore less vibrated by a given impulse.

The monolithic nature of the construction also reduces vibration greatly, and the author is quite satisfied with the above formula (1) for reinforced concrete construction, after having studied the subject with great care for some years.

Now it is clear that if this allowance is adequate for speeds of 60 miles per hour a smaller allowance will do for small speeds.

It would appear reasonable that the live load allowance should depend on the time taken to apply the load. Thus, if one girder 88′ long is loaded in one second by a train travelling at 60 miles an hour, one 176′ long will be in two seconds, and so will one of 88′ by a train travelling only at 30 miles an hour.

It would appear reasonable to apply a speed factor which should make the same reduction in live load allowance when the speed is halved as is affected when the span is doubled. This is done by the author's formula:

$$\text{live load factor} = \left(1+\frac{100}{100+l\frac{60}{S}}\right) \quad . \quad . \quad (2)$$

where $l$ is the span in feet and $S$ the train speed in miles per hour.

This factor gives the following values for different speeds and spans, as shown in Table VI, and Fig. 71.

It may be objected on theoretical grounds that the vibration should vary with the square of the speed and not with the speed but experience hardly bears this out on bridge work. It must be remembered that the vibration is caused chiefly by the following factors:

(1) Unbalanced engines.

(2) Inequalities in track, particularly at rail joints.

(3) Rapidity of loading, with resultant fatigue.

Now high speed engines are always carefully balanced compared to low speed ones, high speed rolling stock is better

sprung, and a high speed track much more carefully laid than a low speed one, and therefore the destructive effect of higher speeds is largely reduced. The results of tests confirm that the vibration does not increase more rapidly than with first power of the speed.

TABLE VI. Live load factors for main longitudinal members of concrete bridges.

| Speed miles per hr. | Span in ft. | | | | | |
|---|---|---|---|---|---|---|
| | 0 | 20 | 40 | 100 | 200 | 300 |
| 0 | — | — | 1 | 1 | 1 | 1 |
| 10 | 2 | 1·45 | 1·29 | 1·14 | 1·08 | 1·05 |
| 20 | 2 | 1·62 | 1·45 | 1·25 | 1·14 | 1·10 |
| 30 | 2 | 1·72 | 1·55 | 1·33 | 1·20 | 1·14 |
| 40 | 2 | 1·77 | 1·62 | 1·40 | 1·25 | 1·18 |
| 50 | 2 | 1·81 | 1·67 | 1·45 | 1·29 | 1·22 |
| 60 | 2 | 1·83 | 1·72 | 1·5 | 1·33 | 1·25 |

*Note.* Cross girders, slabs, ties or struts in main girders carrying cross girders, etc., to be designed for live load factor of two.

The reduction in the factor with increasing span is because the live load will then be on many axles, and probably only one of these will be passing a rail joint at the one moment. Thus on a bridge 40 ft. long, there may be five axles, so if we allow 29 per cent. increase on the total live load (as suggested in the table for 10 miles per hour) we are allowing $5 \times 29$ per cent. $= 145$ per cent. increase on the one axle which is jumping the joint, clearly an ample allowance, and on a 20 ft. span with three axles, we are allowing $3 \times 45$ per cent. $= 135$ per cent. on the wt. of one axle.

The weight and stiffness of concrete bridges prevents any likelihood of serious surging owing to the period of vibration of the bridge coinciding with that of the unbalanced vibration of the engine, or of the knocking at the rail joints, which in steel bridges is often a real danger.

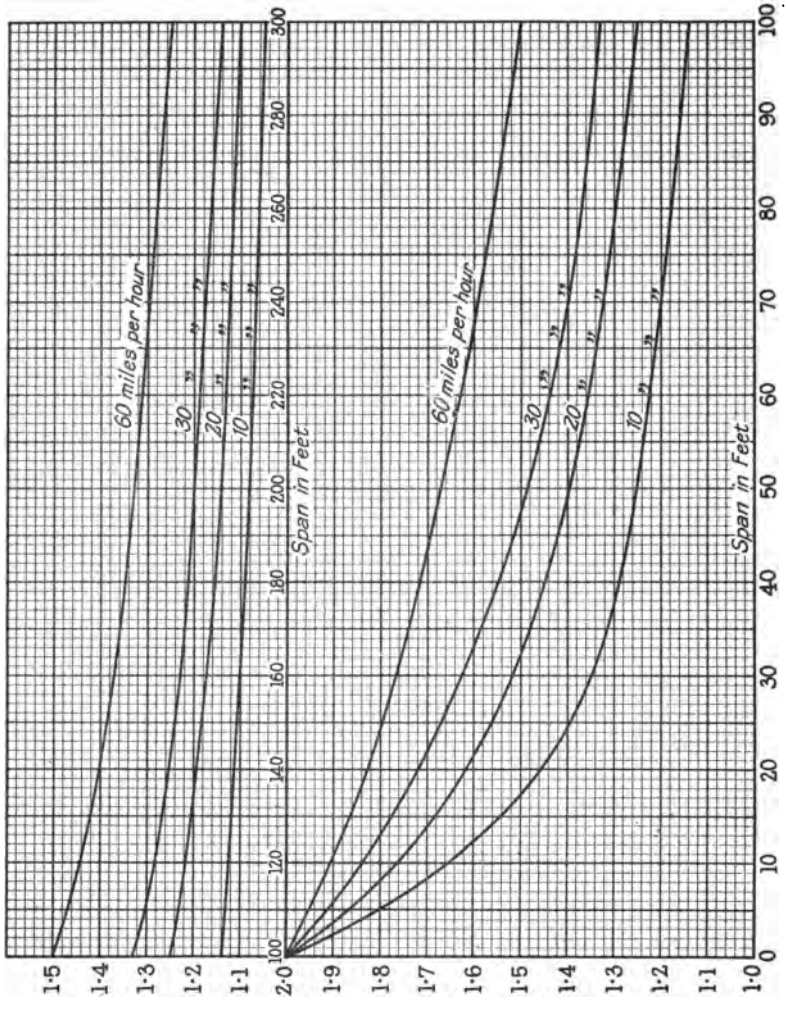



Fig. 71. Live Load Factors for Main Longitudinal Members of Concrete Bridges.

# CHAPTER XV

## SHEAR RESISTANCE OF REINFORCED CONCRETE BEAMS

Since the writing of Vol. I the author has carried out considerable researches on this subject, as a result of which considerable additional information now exists. For a full treatment of this subject, with a description of the experimental work, the reader is referred to *Shearing Resistance of Concrete Beams*, published by *Concrete and Constructional Engineering*, London. This also gives the theoretical justification for the following rules, which is here omitted.

It will be convenient to summarize the results as follows:

(1) A beam will without shear reinforcement resist shear up to

$$s = tba \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where $t$ = safe tensile strength, often taken at 60 lbs./ins.$^2$, $b$ = breadth of beam or rib in T beams, $a$ = depth from centre of tension to centre of compression (generally about $\cdot 88d$).

(2) Beyond this limit, the whole shear must be resisted in other ways, and the tension neglected. It is wrong to design shear reinforcement to resist the difference between the total shear and $tba$.

(3) The other ways referred to in (2) are

($a$) vertical stirrups,

($b$) bent-up bars,

($c$) direct inclined compressions.

(4) The action of stirrups involves the formation of compression forces in the concrete at an angle of about 45°, between the bottom of stirrups near the support and the top of stirrups nearer mid span. These forces are called "indirect compressions" because they do not go direct from load to support (see later). As a rule, the resistance to shear of stirrups is limited by the resistance of the steel stirrups, and is given by

$$S = \frac{Ata}{l} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where $A$ = total area of cross section of different members of

one stirrup, $t$ = safe tensile stress in stirrups, $a$ = radius arm (distance from centre of compression to centre of tension, generally about $\cdot 88d$), $l$ = spacing of stirrups. $l$ must not exceed $a$ and should generally be much less.

If the reinforcement by way of stirrups is carried beyond a certain point, failure will occur by crushing of the concrete.

This limiting value is

$$S_{\text{max.}} = \frac{abc}{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where $a$ = radius arm as before, $b$ = breadth of beam or rib of T beam, $c$ = safe compressive stress in concrete.

Close spacing of stirrups, good security against slipping, good distribution with large bars top and bottom are necessary for this value to be approached without reduction of safety factor.

Where the aggregate width of the bars engaged by the stirrups in the top or bottom (whichever is least) is less than

one-third $b$,

then three times the width of bars should be used instead of $b$ in formula (3).

It will be seen that the value of shear per square inch of beam section may therefore approach 300 lbs./ins.$^2$ in favourable conditions when all shear is taken by a stirrup system.

(5) The action of bent-up bars also involves the formation of indirect compressions which tend to lie at 45°, but the angle depends on the distribution of bars.

If a bar is inclined at an angle $\theta$ to the horizontal, has an area $A$, and is stressed to a safe stress $t$, then its resistance to shear is the vertical component, and

$$S = t \,.\, A \,.\, \sin\theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Bent-up bars may be so disposed as to form a single or a double system.

If a beam is considered as divided up into square panels, equal in length to the radius arm $a$, and a bent-up bar provided across the diagonal of alternate panels, then an "indirect compression" at 45° will be produced in the remaining alternate panels, the vertical component of which will give the same shear resistance as the vertical component of the inclined tension in the panels with the bent-up bars.

Such a system is called a "*single*" *system.*

If now we add a bent-up bar across the diagonals of the panels which before did not contain them, then a vertical section across these panels will intersect a diagonal tension and a diagonal compression, each of which gives a vertical component, or shear resistance,

$$S = t \,.\, A \,.\, \sin\theta,$$

and hence the shear resistance will be

$$S = 2t \,.\, A \,.\, \sin\theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Such a system is called a *double system.*

If we take $t = 16{,}000$ lbs./ins.$^2$ or similar stresses, two precautions are necessary. One is to see that the bar at the top of the diagonal is fixed well enough to develop its full stress at this point. This generally involves running it some distance, say 30 to 40 times its diameter along the boom, and providing an efficient hook (see Fig. 39, Vol. I). The other is to have all bends so gradual that the concrete is not overstressed at the bend. We may allow $3c$ as the stress on the concrete as a bend, provided the width of the beam or rib is not less than three times the diameter of the bars bent up at one place. This means the minimum radius allowable is

$$r_{\text{min.}} = 7d \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where $d$ is the diameter of bent-up bars. [This is on the basis of $c = 600$ lbs./ins.$^2$ in the concrete, and $t = 16{,}000$ lbs./ins.$^2$ in the steel.]

With a smaller radius, a lesser value of the safe shear should be adopted, as also when the fixing beyond the bend-up is insufficient to develop the full strength.

When the strength of bent-up bars is progressively increased, a stage is eventually reached when the concrete fails under the *inclined compressions* before developing the full strength of the bars.

This limiting value for a *single system*

$$S_{\text{max.}} = \frac{abc}{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

when $\theta$ is 45°, and is

$$S_{\text{max.}} = a \,.\, b \,.\, c \,.\, \cos\theta \,.\, \sin\theta \quad . \quad . \quad . \quad . \quad (8)$$

for any lesser angle.

In either of these formulae, if the aggregate width of bent-up bars at any place is less than one-third $b$, the width of beam or rib of T beam, three times such aggregate width should be substituted for $b$ in formulae (7) or (8).

For a double system, the limiting values are twice as great.

It will be seen that with $c = 600$ it is possible with a well-designed double system to approach

$$\frac{S}{ba} = 600 \text{ lbs. per square inch.}$$

The L.C.C. regulations for reinforced concrete frame buildings state that under no circumstances shall

$$\frac{S}{ba} \text{ exceed } 180 \text{ lbs. per square inch}$$

for ordinary 1, 2, 4 concrete.

The writer has constructed and tested beams giving

$$\frac{S}{ba} = \text{ultimate,}$$

showing that this L.C.C. regulation needs amendment and is unnecessarily onerous.

It is serious, because it sometimes means concrete beams need to be so big as to be ruled out on architectural grounds in important buildings, and steel construction substituted.

(6) In addition to shear resistances produced by systems of stirrups or of bent-up bars, there are shear resistances due to *direct compressions*.

A direct compression occurs when the load is applied to the top of a beam, and may be transmitted to the support *direct* by an inclined compression without needing to be transmitted in several stages, and lifted after each stage from the bottom to the top flange by stirrups or bent-up bars.

(7) For direct compressions to exist in beams with free ends, there must be steel rods in the bottom flange continued through to the support and provided there with ample anchorage. This generally requires it to be carried on about 30 or $40d$* beyond the support and provided with an efficient hook.

* Where $d$ is the diameter of the bar.

In such beams, not provided with top reinforcement, the value of such shear resistance may be taken as

$$S = \frac{3a \,.\, b \,.\, c \,.\, d_1}{kl} \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

where $d_1$ = distance of centre of tension from bottom of beam, $l$ = span, $k$ = a factor depending on the distribution of loads.

For single point load at mid-span, $k = 1$
„ 2 point loads at third points, $k = \frac{2}{3}$
„ uniform load ... ... $k = \frac{1}{2}$

If we take $c = 600$

and $d_1 = \cdot 12d$

$a = \cdot 88d$

$h = d + d_1 = 1 \cdot 12d$

formula (9) resolves to

$$S = \frac{150bh^2}{kl} \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

To justify this value the steel carried through to the support must not be less than ·675 per cent. of $bd$.

With steel top and bottom, the safe shear may reach

$$S = \frac{225bh^2}{kl} \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

This requires 1·55 per cent. of $bd$ steel in lower flange and ·675 per cent. in the upper flange.

(8) Formulae (10) and (11) also apply to continuous beams, provided these contain steel running along both flanges the whole length.

The steel required [·675 per cent. for (9) and (10) and 1·55 per cent. for (11)] is then to be placed, not in the lower flange, but divided between bottom and top flanges in the ratio of the moments at centre and support.

Thus if $M$ = total free moment, then steel in lower flange required

$$= \frac{\cdot 675bd}{100} \times \frac{M_c}{M} \text{ for (9) and (10)}$$

and

$$\frac{1 \cdot 55bd}{100} \times \frac{M_c}{M} \text{ for (11)}$$

and steel in top flange required

$$= \frac{\cdot 675bd}{100} \times \frac{M_s}{M} \text{ for (9) and (10)}$$

$$= \frac{1\cdot 55bd}{100} \times \frac{M_s}{M} \text{ for (11)}$$

where $M_c$ = max. centre moment under worst conditions

$M_s$ = support moment " "

If the steel required by these formulae is not provided, the shear resistance by direct compression must be reduced in the ratio of what is provided to what is required.

(9) The values of shear by direct compression are given by the following table and in Fig. 72.

TABLE VII.

End shear by direct inclined compressions for $c = 600$.

| $\frac{\text{Factor} \times \text{Span}}{\text{radius arm}} = \frac{kl}{a}$ | Safe shear resistance | |
|---|---|---|
| | Steel at least ·675 % of $bd$ placed as described [formula 10] | Steel both sides and at least 1·55 % as described [formula 11] |
| 2 | 122 $a \cdot b$ | 183 $a \cdot b$ |
| 4 | 61 | 91·5 |
| 5 | 49 | 73·5 |
| 10 | 24·5 | 36·75 |
| 15 | 16·2 | 24·3 |
| 20 | 12·2 | 18·3 |

(10) It may be shown that rectangular beams not reinforced in compression (such as slabs) having their full area of steel continued to the support and secured there will *always* have an adequate resistance to shear by direct inclined compression.

(11) The tabulated values for the direct inclined compression are not constant along the span, but have the values given at the end only. They fall off towards mid-span in direct proportion as the shear, in accordance with the shear diagram.

Thus, for uniform load, it falls off uniformly to the centre, for a single point load remains constant to mid-span, and for other cases, falls in accordance with the shear diagram.

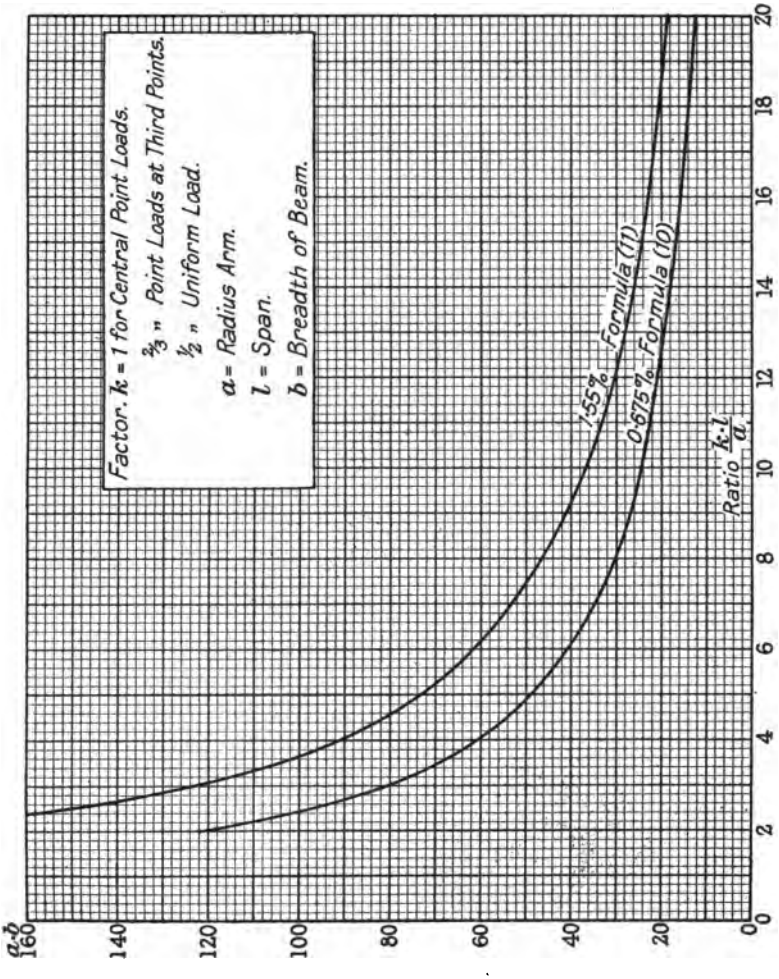



Fig. 72. Shear Resistance by Direct Inclined Compression.

(12) It is perfectly justifiable to calculate separately and add together resistances due to stirrups and bent-up bars. The only proviso is that the concrete may not be overstressed.

If $\frac{S}{ab}$ does not exceed $\frac{c}{3}$ lbs./ins.$^2$

this will not be so. If this is exceeded, the concrete stress needs investigation.

With *stirrups* and a *single system* of bent-up bars in combination,

$\frac{S}{ab}$ may never exceed $\frac{c}{2}$ lbs./ins.$^2$

With *stirrups* and a *double system* of bent-up bars in combination

$$\frac{S}{ab} \text{ may never exceed } c\left[\frac{\frac{S_1}{2}+S_2}{S_1+S_2}\right]$$

where $S_1$ = shear by stirrups,

$S_2$ = shear by double system of bent-up bars.

(13) Direct inclined compression may be added to the other resistances, but not in full. The other resistances have partly stressed the concrete, and only the remaining available safe stress can be used for the direct inclined compressions.

The concrete stress by a stirrup system is given by

$$c_1 = \frac{2S_1}{ab} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

where $S_1$ = total shear taken by stirrups.

The concrete stress by a single bent-up bar system is

$$c_2 = \frac{S_2}{(\cos\theta \,.\, \sin\theta)\; a \,.\, b} \quad . \quad . \quad . \quad . \quad (13)$$

where $S_2$ = total shear taken by single bent-up system, and

$\theta$ = inclination of inclined compression and tension to the horizontal.

When $\theta = 45°$, this reduces to

$$c_2 = \frac{2S_2}{a \,.\, b} \quad . \quad . \quad . \quad . \quad . \quad . \quad (13')$$

For a double system

$$c_2 = \frac{S_2}{2\,(\cos\theta \,.\, \sin\theta)\,ab} \quad . \quad . \quad . \quad . \quad (14)$$

When $\theta = 45°$, this reduces to

$$c_2 = \frac{S_2}{ab} \quad . \quad . \quad . \quad . \quad . \quad . \quad (14')$$

Add $c_1$ to $c_2$. The sum must not exceed $c$ (the safe stress).

The difference $c - (c_1 + c_2)$ is available for direct inclined compressions, and the tabulated values may be adopted, multiplied by the factor

$$\frac{c - (c_1 + c_2)}{c}$$

and added to the other shear resistances. If the shear under consideration is not at the end the inclined compression resistance must be reduced as the total shear is reduced.

The above is not an accurate treatment, owing to the possibly different inclinations of the different inclined compressions. It is safe, and the reader is referred to the fuller treatise on shear mentioned early in this chapter.

# APPENDIX I

## ALIGNMENT DIAGRAMS FOR REINFORCED CONCRETE BEAMS

Engineers experienced in the use of alignment diagrams have often found them extremely convenient and therefore a series of such diagrams is added in Appendix I.

These diagrams are applicable to rectangular reinforced concrete beams, singly reinforced, i.e. with reinforcement on the tension side only, and subjected only to bending without direct tension or compression. Examination of the diagram shows that the left-hand scale gives the tensile stresses in the steel, the centre scale compressive stresses in the concrete and the right-hand scale gives on the right-hand side the percentage of steel and on the left-hand side the lever arm between centre of tension and centre of compression of the resisting forces, these being, as was fully explained in *Reinforced Concrete Design*, Vol. I, Ch. II, definitely fixed when the percentage is determined.

To use the diagram all that is necessary is to place a straight edge on the diagram so that it intercepts all three scales, the values of which will then be strictly coexistent. Thus, for example, suppose on a certain beam which has a percentage of ·8 it is found that a given bending moment produces a stress of 16,500 and it is desired to know what will be the compressive stress in the concrete, join the 16,500 figure on the left-hand scale to the ·8 figure on the right-hand scale and the intersection of the centre scale will show the concrete stress to be 687. Similarly, if it is desired to know what percentage of steel is necessary to give 14,000 tensile stress in the steel and 500 compressive stress in the concrete, join 14,000 on the left-hand scale to 500 on the centre scale and produce it till it cuts the right-hand scale when it will be seen that ·62 per cent. is required.

On the right-hand scale it would have been convenient if there had also been room to give the depth of the neutral axis which clearly, as fully explained in Vol. I, Ch. II, depends on

the percentage of steel only, but this may readily be obtained from the lever arm by subtracting the lever arm from 1 and multiplying the resultant by 3. For example, if the lever arm works out at ·9 then the depth of the neutral axis will be

$$n_1 = (1 - \cdot 9) \times 3 = \cdot 3$$

This figure is, of course, the ratio of the depth of the neutral axis from the compressed edge to the depth of the steel from the compressed edge, and therefore if the depth of the neutral axis is required in inches the figure needs to be multiplied by the depth of the beam to the steel in inches.

So far the use of the alignment diagram has been absolutely accurate apart from graphical errors and no approximations have been made.

Reference to the diagram will show a fourth scale for the moment of resistance $\frac{R}{bd^2}$, the use of which is exactly similar to that of the other scales, but it should be noted that though the use of this particular scale does involve certain approximations it does not cause errors exceeding 1 per cent. within the range of the diagram.

The alignment diagram may be used, if desired, for cases where compression steel is also employed by finding the increased width of singly reinforced beams which would give the same stresses, as was fully explained in *Reinforced Concrete Design*, Vol. I, Ch. II, pp. 42 and 43. The diagram may also be used for T beams in those cases where the neutral axis lies above the bottom of the slab and with sufficient approximation if it lies only very slightly below it.

As regards the convenience of these diagrams there are cases where it is unquestionably very convenient but the author's experience has been that in practice it proves to be not quite so useful as one would at first imagine. It is only applicable for simple bending in singly reinforced members (except where much auxiliary work is done to render it applicable) and for such simple cases the designer generally works to definite stress ratios in which the various constants are readily memorised, and therefore no diagram is necessary, and in more complicated cases, as where compression steel is

used, or in the case of T beams, the methods given in *Reinforced Concrete Design*, Vol. I, the author finds as easy in practice, besides being free from the difficulty that with the alignment diagram one frequently wishes to go above or below the scales and has ultimately to refer to the original method.

However, there is no doubt that for certain simple cases and particularly to certain individuals the diagram is often a great convenience which fully justifies its inclusion.

The author's thanks are due to Messrs F. P. Watson and G. C. Wingfield of the Egyptian Public Works Ministry who first published this diagram and were kind enough to indicate that they would have no objection to its inclusion. They also published a diagram for use in T beams which was extremely ingenious but which the author did not include as on the whole he found it more complicated than the use of the ordinary methods.

UNIV. OF
CALIFORNIA

# APPENDIX II

## A STUDY OF MOMENTS IN BEAMS, ALLOWING FOR STIFFNESS OF COLUMNS

*NOTE.* In the author's opinion, only the most skilful and experienced engineers should consider this treatment—for all others it is too dangerous.

Even for them it is too cumbersome for general use in practice except in special cases.

Nevertheless the author attaches the utmost importance to it, since it puts within reach of all, for the first time, the absolutely accurate treatment of moments, taking live and dead load and stiffness of columns into account, in a manner which to the author appears eminently clear and practical.

In Part I it was shown in great detail how to design for moments in beams neglecting stiffness of columns, and actual moment diagrams are given, which may safely and easily be used in practice.

These diagrams are always on the side of safety with the sole exception—and this is important—that the moments between end beams and end columns are not given on them, and these *must* be separately calculated and allowed for. This generally means providing top steel in the beam as the joint. The factors given in Part II enable this to be readily calculated.

In the following analysis the stiffness of columns is allowed for in the calculation of moments *throughout* the beam. For convenience, the different spans of beams are assumed to have the same moment of inertia, as are also the different columns. In good design this is generally approximately true.

*Case I.* **Beam of one span monolithic with two columns.**

In Fig. 64, p. 126, and Table III, p. 125, were given values of $F_{14}$ which, multiplied by $-\frac{Wl}{12}$ gives the moment at the end of the beam.

Clearly the moment at midspan is $\frac{Wl}{8}$ less this moment and is therefore easily calculated.

Actual moment curves for different ratios of $\frac{KC}{B}$ are given in Fig. 73.

Intermediate values can easily be obtained approximately by inspection or actually drawn, using the correct value of $F_{14}$ from the curve Fig. 64.

Note that for $\frac{KC}{B} = 1$, the centre moment is reduced to 78 % of $\frac{Wl}{8}$, and for $\frac{KC}{B} = 6$, to 50 % of $\frac{Wl}{8}$.

*Case II.* **Beam of two spans, monolithic with three supports.**

This is much more complicated than Case I nor are such large reductions achieved.

Consider first the *dead load* separately (the same curve applies of course, when both spans are fully loaded).

From symmetry the moment in the centre column is zero.

The moment in the outer column was given, for this case, by factor $F_{15}$ (dead load case) multiplied by $-\frac{Wl}{12}$, $F_{15}$ being given in Fig. 65, p. 128, and the right-hand column of Table IV, p. 138. The moment increases gradually from 0 for $\frac{KC}{B} = 0$ to $-\frac{Wl}{12}$ for $\frac{KC}{B} = \infty$.

The moment in the beam at the centre support clearly is $-\frac{Wl}{8}$ for $\frac{KC}{B} = 0$ and $-\frac{Wl}{12}$ for $\frac{KC}{B} = \infty$, intermediate values being obtained as follows.

In Vol. I, p. 275, this is given in equation (3) thus

$$M_2 = -\frac{Wl}{12} + 2EB\alpha_1 + 4EB\alpha_2 \quad . \quad . \quad . \quad (3)$$

Remembering that $\alpha_2 =$ zero

and $M_1 = KCE\alpha_1$

whence $E\alpha_1 = \frac{M_1}{KC}$

$$= F_{15} \times \frac{1}{KC} \times \left(-\frac{Wl}{12}\right)$$

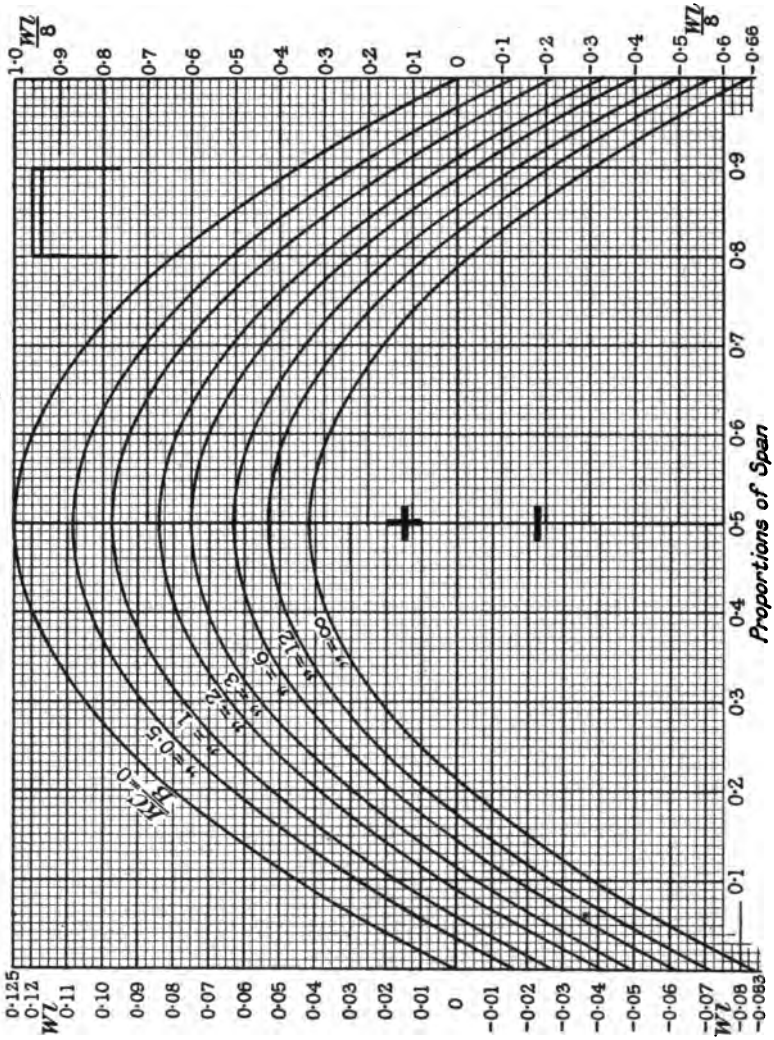



Fig. 73. Moments in Beam of one Span Monolithic with Columns.

this reduces to

$$M_2 = -\frac{Wl}{12}\left(1+\frac{2B}{KC}\cdot F_{15}\right)$$
$$= F_{19}\times\left(-\frac{Wl}{12}\right).$$

Values of $M_2$ or $F_{19}$ are therefore easily calculated from $F_{15}$ and are given in the following table:

TABLE I. Of Factor $F_{19}$.

| $\frac{KC}{B}$ | $F_{19}$ |
|---|---|
| 0 | 1·5 |
| ·1 | 1·492 |
| ·2 | 1·48 |
| ·5 | 1·444 |
| 1 | 1·39 |
| 2 | 1·333 |
| 5 | 1·222 |
| 10 | 1·143 |
| 20 | 1·083 |
| 30 | 1·059 |
| ∞ | 1 |

Values of $F_{15}$ and $F_{19}$ are given in Fig. 74.

With these values known, there is of course no difficulty in drawing the actual moment diagrams, some of which are given in Fig. 75.

The *live load* is unfortunately more complicated, since with one span loaded and the next unloaded, there are five end moments needing evaluation (see Fig. 76).

Of these $F_{15}$ for the moment in the end column of the loaded bay was given on pp. 128 and 138 (left-hand column).

The live-load moment in the centre column was given by $F_{17}$, pp. 140 and 143.

The moment in the end column of the unloaded bay is obtained as follows.

From Vol. I, p. 275, equation under number (10),

$$E\alpha_3 = \frac{W_2 l_2^2}{12}\left(\frac{1}{K_3C_3+4B_2}\right) - 2EB_2\alpha_3\left(\frac{1}{K_3C_3+4B_2}\right).$$

Now first term disappears since $W_2$ = zero and equation becomes

$$E\alpha_3 = -E\alpha_2\cdot\frac{2B}{KC+4B}.$$

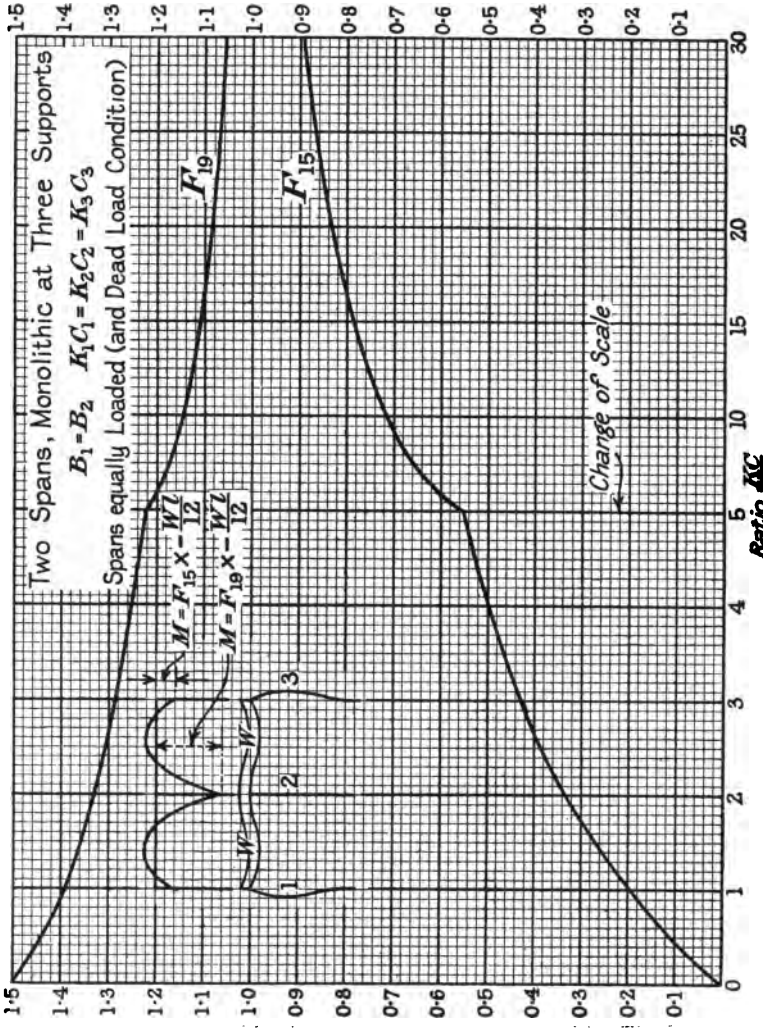

Two Spans, Monolithic at Three Supports
$B_1 = B_2$ $K_1C_1 = K_2C_2 = K_3C_3$
Spans equally Loaded (and Dead Load Condition)
$M = F_{15} \times \frac{WL}{12}$
$M = F_{19} \times \frac{WL}{12}$
$F_{19}$
$F_{15}$
Change of Scale
Ratio $\frac{KC}{B}$
1·5 1·4 1·3 1·2 1·1 1·0 0·9 0·8 0·7 0·6 0·5 0·4 0·3 0·2 0·1
0 1 2 3 4 5 10 15 20 25 30
1 2 3
WL WL

Fig. 74. Two Spans Monolithic. Factors $F_{15}$ and $F_{19}$. Dead Load.

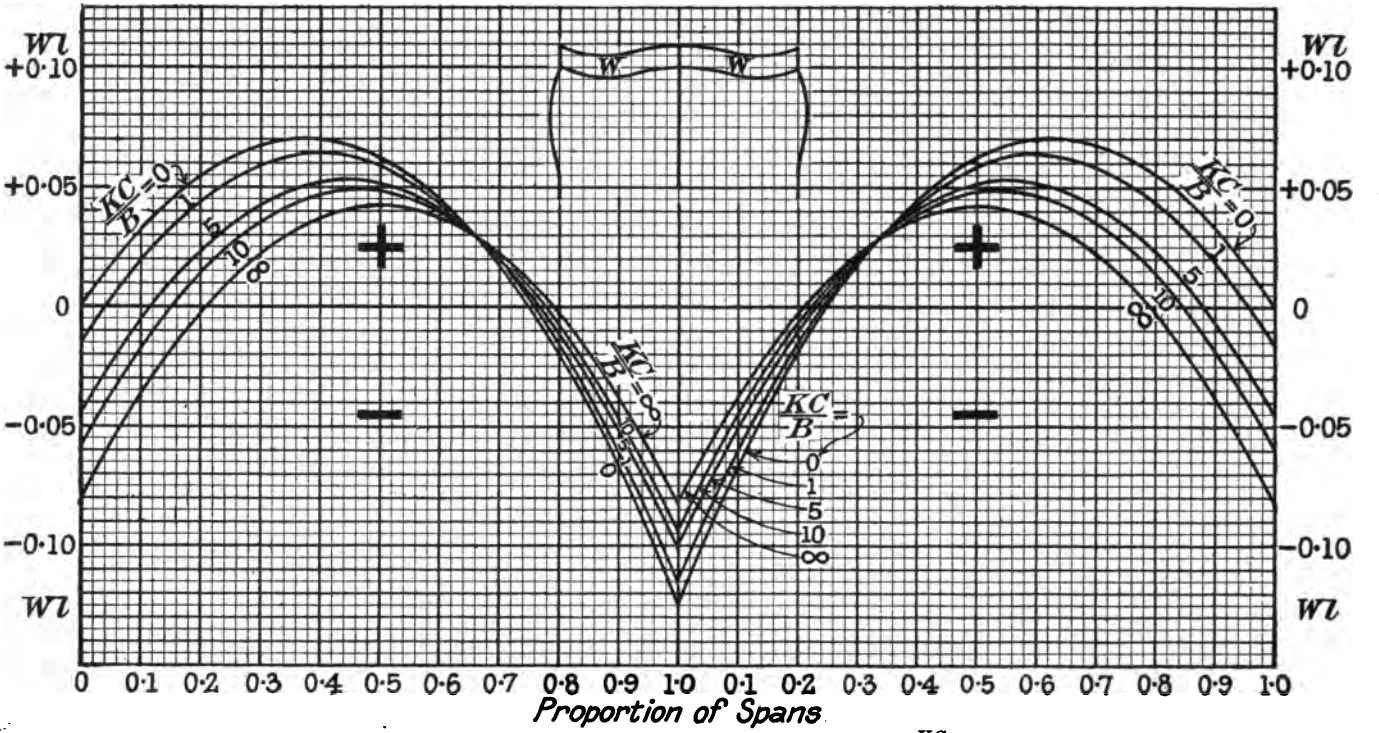



Fig. 75. Moments in two Spans Monolithic with three supports for different ratios of $\frac{KC}{B}$. Dead load on both bays loaded.

The $KCE\alpha_2 =$ moment on centre column

$$= F_{19} \cdot \frac{Wl}{12},$$

$$\therefore E\alpha_2 = \frac{F_{19}}{KC} \cdot \frac{Wl}{12},$$

$\therefore$ Moment in unloaded end column ($M_4$, Fig. 76)

$$\begin{aligned} M_4 &= E\alpha_3 KC \\ &= -E\alpha_2 \cdot \frac{2B}{KC + 4B} \cdot KC \\ &= -\frac{F_{19}}{KC} \cdot \frac{Wl}{12} \cdot \frac{2B}{KC + 4B} \cdot KC \\ &= F_{19} \cdot \frac{2B}{KC + 4B} \cdot \frac{Wl}{12} \\ &= F_{18} \cdot \frac{Wl}{12}. \end{aligned}$$

Values of $F_{18}$ are thus readily calculated from $F_{19}$ and are given in Fig. 76 and Table II.

The moment in the loaded span at the centre support ($M_2$) is given thus

$$M_2 = -\frac{Wl}{12} + 2EB\alpha_1 + 4EB\alpha_2$$

(Equation (3), Vol. I, p. 275)

$$= -\frac{Wl}{12} + \frac{2B}{KC} \cdot \frac{Wl}{12} \cdot F_{15} - \frac{4B}{KC} \cdot \frac{Wl}{12} \cdot F_{17}.$$

The last negative sign is owing to $\alpha_1$ and $\alpha_2$ being of opposite sign. The values of $F_{15}$ are the live load ones. This reduces to

$$M_2 = -\frac{Wl}{12}\left(1 + \frac{2B}{KC} \cdot F_{15} - \frac{4B}{KC} \cdot F_{17}\right) = -\frac{Wl}{12} \times F_{20}.$$

Values are calculated and given in Fig. 76 and Table II.

We are left with the moment in the unloaded span at the centre support and clearly

$$\begin{aligned} M_3 &= M_2 - \text{column moment} \\ &= -\frac{Wl}{12}(F_{20} - F_{17}) \\ &= -\frac{Wl}{12} \cdot F_{21}. \end{aligned}$$

Values are given in Fig. 76 and Table II.

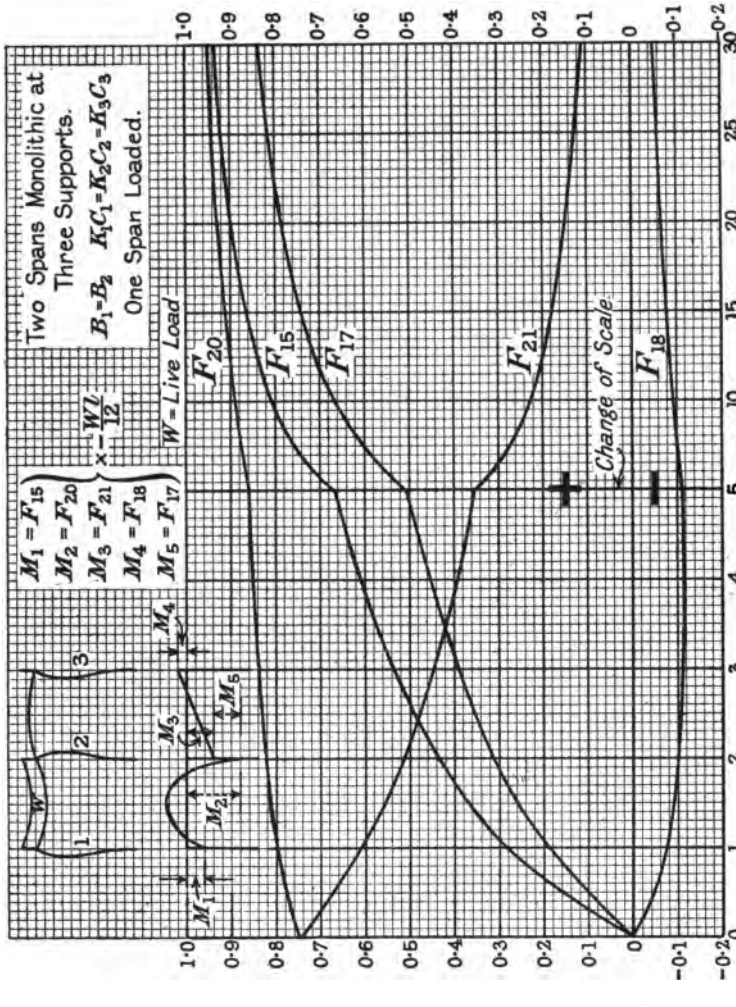



Fig. 76. Moments in two Spans Monolithic. Values of Live Load. Factors $F_{15}$, $F_{17}$, $F_{18}$, $F_{20}$, $F_{21}$.

TABLE II. Factors for live load curves, two spans monolithic (see Fig. 76).

| $\frac{KC}{B}$ | $F_{15}$ | $F_{20}$ | $F_{21}$ | $F_{18}$ | $F_{17}$ |
|---|---|---|---|---|---|
| 0 | 0 | ·75 | ·75 | −·0 | 0 |
| ·1 | ·036 | ·755 | ·731 | −·0117 | ·0241 |
| ·2 | ·069 | ·76 | ·713 | −·0223 | ·0469 |
| ·5 | ·157 | ·78 | ·673 | −·0478 | ·1075 |
| 1 | ·281 | ·80 | ·611 | −·0756 | ·189 |
| 2 | ·435 | ·82 | ·513 | −·1025 | ·307 |
| 5 | ·666 | ·86 | ·355 | −·1125 | ·505 |
| 10 | ·805 | ·89 | ·223 | −·095 | ·666 |
| 20 | ·9 | ·93 | ·147 | −·065 | ·783 |
| 30 | ·937 | ·95 | ·11 | −·0495 | ·84 |
| ∞ | 1·0 | 1·0 | 0 | 0 | 1·0 |

From these values there is no difficulty in drawing the live load for any value of $\frac{KC}{B}$.

Some such curves are given in Fig. 77, which show very clearly the effect of column stiffness. Moments for intermediate values of $\frac{KC}{B}$ can be estimated or separately obtained from values in Fig. 76.

The positive moment curves occur with one span loaded, as do the negative moment curves. These, for any value of $\frac{KC}{B}$, do not intersect at a point on the centre support, owing to the moment in the centre column with this condition of loading.

The positive moment curves near the centre support are not continued below the zero line. A worse negative moment exists, for points near the centre support, for the condition of both spans fully loaded (as was the case in the treatment in Part I) and therefore the curves from Fig. 75 are reproduced here to supply the negative cusps, the principle being, of course, to use at any section that moment curve giving either the worst positive or worst negative moment, and ignoring the others. In these respects we follow the treatment of Part I.

In any actual case, it would be necessary to apply the appropriate curve in Fig. 74 to the dead load, and add the values (having due regard to sign) to those obtained from appropriate curves in Fig. 76 applied to the live load, both positive and negative.

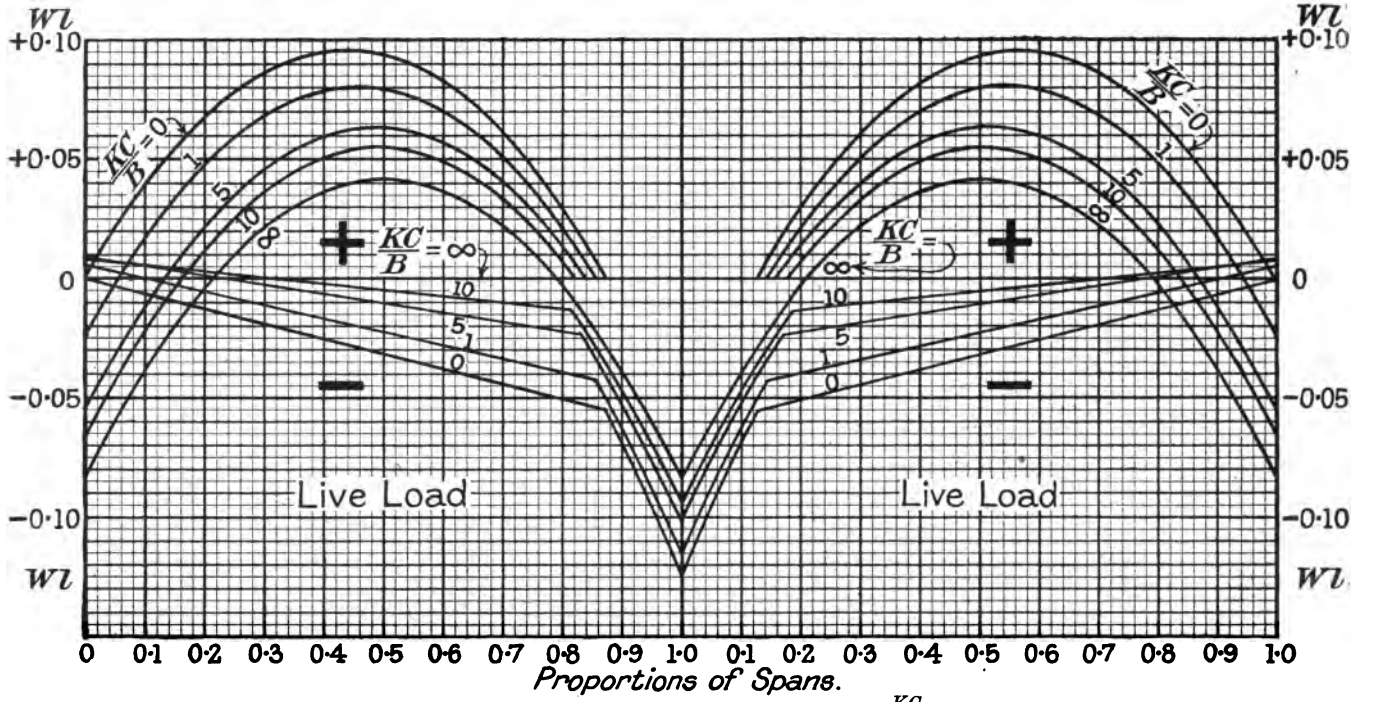



Fig. 77. Moments in two Spans Monolithic with three supports for different ratios of $\frac{KC}{B}$. Maximum Live Load Moments.

*Case III.* **Many spans, monolithic with all supports.**

In this we follow the general treatment for the same case as in Part I in considering (1) an interior span,

(2) an end span.

All interior spans are sensibly alike, and we take one sufficiently far from the end not to be affected by the actual number of spans as our standard.

For the end span, we consider the end one of *three* spans, for the same reasons as before (see pp. 49–51).

As before, we must consider dead and live load separately.

*Dead load.*

The dead load moment curve for *interior spans* is not affected by the stiffness of columns and one curve does for all ratios of $\frac{KC}{B}$.

The reason for this is of course that the dead load applying equally to the various spans does not bend the columns and therefore produces no moments in them (see Fig. 78).

With the *end span* it is different. With slender columns $\left(\frac{KC}{B}=0\right)$ the moment curve is a skew parabola erected on a base line joining $M=0$ at the outer support to $M=-\frac{Wl}{10}$ at the inner support. As the stiffness of columns increases, the moment at the outer support greatly increases and at the inner support slightly decreases, both reaching $-\frac{Wl}{12}$ in the limiting case $\left(\frac{KC}{B}=\infty\right)$.

The moment at the outer support is given by $F_{15}\cdot\left(-\frac{Wl}{12}\right)$, $F_{15}$ being tabulated on p. 138 and Figs. 64 and 65 (the *dead load* figures to be used).

The moment at the inner support is given by the following:

| $\frac{KC}{B}$ | Moment |
|---|---|
| 0 | ·10 $Wl$ |
| 1 | ·097 |
| 5 | ·091 |
| 10 | ·088 |
| ∞ | ·083 |

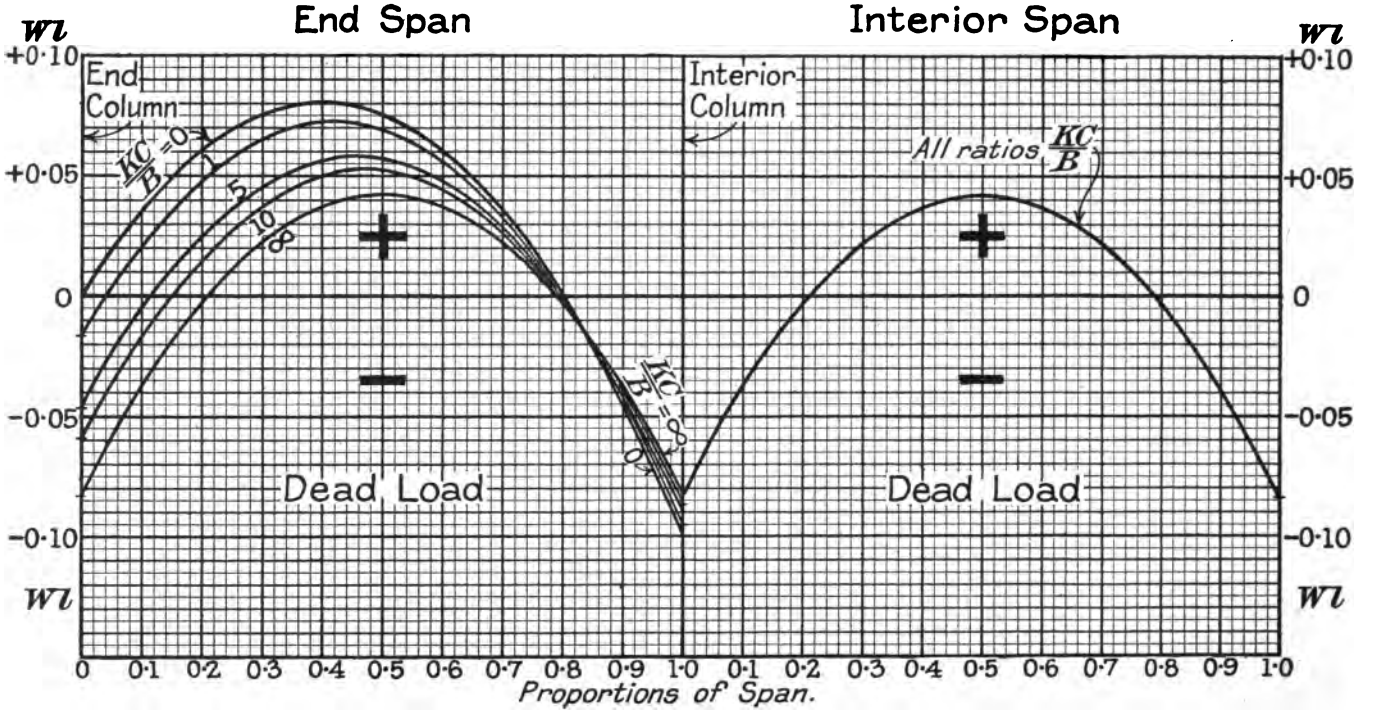



Fig. 78. Moments in End and Interior Spans Monolithic with all support for different ratios of $\frac{KC}{B}$. Dead Load.

From this information the curves are easily drawn and some are given in Fig. 78.

*Live load.*

*Interior spans.* As in the treatment in Part I the worst cases in all sections of beam are covered by completely designing for the two conditions:

(*a*) Alternate spans loaded.
(*b*) Adjacent spans loaded.

*Condition* (*a*). *Alternate spans loaded.* The first of these was investigated in Vol. I, Appendix I, 10, pp. 283 and 284, with the result that expressions were obtained for the slopes at the supports, which, multiplied by the general equation $M = KCEa$, gave

$$\text{moment in column } M_1 = -\frac{Wl}{12}\left(\frac{KC}{KC+4B}\right) \quad . \quad . \quad (1)$$

$$= -\frac{Wl}{12} \cdot F_7$$

where $W$ = live load only.

Values of $F_7$ were tabulated in Chap. XII and given in Figs. 69 and 70, but it is convenient to plot it also in Fig. 79, as it affects our present problem also.

The moment at mid-span is

$$M_2 = \frac{W_l l}{24} + \frac{W_l l}{12} \cdot \frac{2B}{KC+4B} \quad . \quad . \quad . \quad . \quad (2)$$

Neglecting dead load (moments from which will be calculated separately as in Part I) this may be written

$$M_2 = +\frac{Wl}{12}\left(\frac{1}{2} + \frac{2B}{KC+4B}\right) \quad . \quad . \quad . \quad (2a)$$

$$= +\frac{Wl}{12} \cdot F_8$$

where $F_8$ is the expression in the brackets.

Negative moment due to live load in the unloaded span is the moment at the support on the loaded span less that part taken by the column.

Hence negative moment in unloaded panel

$$M_3 = M_2 - \frac{Wl}{8} + M_1$$

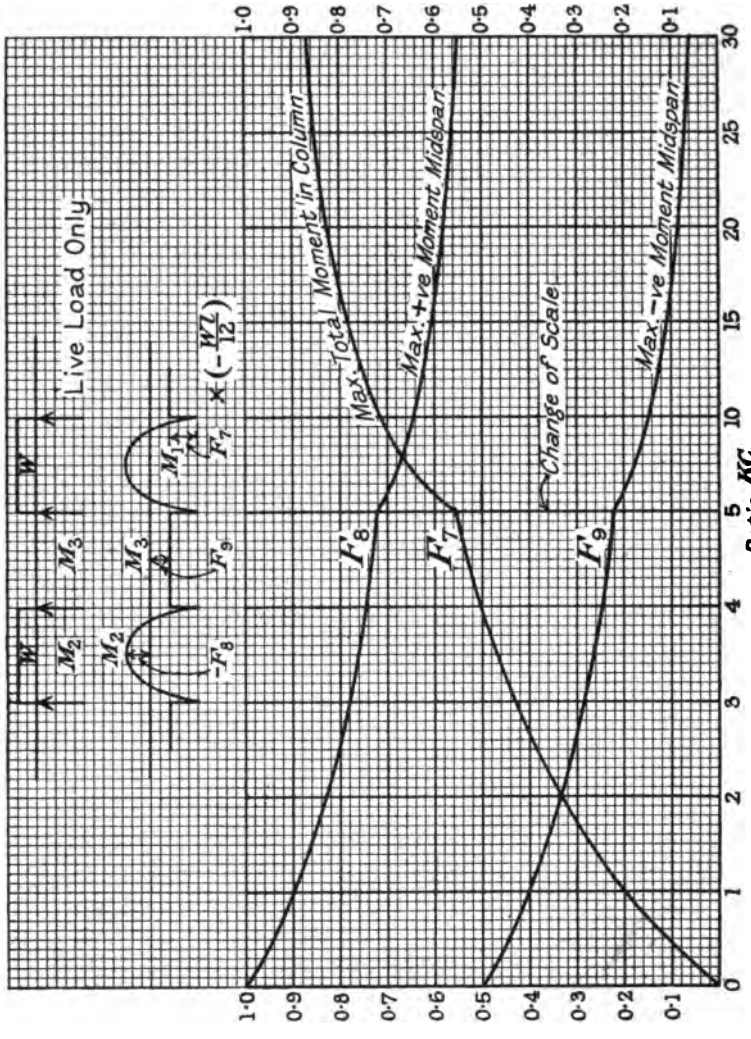



Fig. 79. Factors $F_7$, $F_8$, $F_9$ of maximum moment in Columns and in Beams at Midspan.

Substituting values for $M_2$ and $M_1$ and reducing

$$M_3 = -\frac{Wl}{12}\left\{1 - \frac{2\,(KC+B)}{KC+4B}\right\} \quad . \quad . \quad . \quad . \quad (3)$$

$$. = -\frac{Wl}{12} \cdot F_9$$

where $F_9$ is the expression in the brackets.

Tabulated values of $F_7$, $F_8$ and $F_9$ for different values of $\frac{KC}{B}$ are as follows:

TABLE III.

| $\frac{KC}{B}$ | $F_7$ | $F_8$ | $F_9$ |
|---|---|---|---|
| 0 | 0 | 1 | ·5 |
| ·5 | ·111 | ·942 | ·444 |
| 1·0 | ·2 | ·9 | ·4 |
| 2 | ·333 | ·833 | ·333 |
| 5 | ·555 | ·723 | ·222 |
| 10 | ·714 | ·642 | ·143 |
| 20 | ·830 | ·583 | ·083 |
| 30 | ·870 | ·555 | ·059 |
| ∞ | 1·0 | ·5 | 0 |

These are all plotted in Fig. 79. It will be seen that with slender columns, $\left(\frac{KC}{B} = 0\right)$, the moment in the column is 0, $[F_7 = 0]$, the positive moment at midspan is $+\frac{Wl}{12}[F_8 = 1]$ and the negative moment in the unloaded span is

$$-\frac{Wl}{24}\left[= -\frac{Wl}{12} \times F_9,\ F_9 = ·5\right].$$

On the other hand, with very stiff columns $\left[\frac{KC}{B} = \infty\right]$, the moment in the column is $\frac{Wl}{12}$, the positive moment at midspan of loaded span is $\frac{Wl}{24}$ and the negative moment at midspan of the unloaded span is 0.

These limiting values could have been obtained more simply, but in practice they do not occur, and it is the intermediate values that interest us and these may be taken from the curves.

It will be seen that for values of $\frac{KC}{B} = 5$, the reduction of positive centre moment is 28 %, and of negative centre moment is 56 % from values when stiffness of columns is neglected.

Our moment curves are however not complete yet, because we have not considered the case of adjacent spans loaded, which gives greater negative moments at the supports and is therefore used for the cusps in the moment diagram at the supports, exactly as we did in Part I. We will now proceed to evaluate these cusps.

*Condition (b). Adjacent spans loaded.*

This case was worked out, to a point, in Vol. I, Appendix I, 11, pp. 285 and 286.

To draw the moment diagram completely we now have the four moments indicated in the small figure in Fig. 80 to calculate.

From the formulae in Appendix I, 11 referred to, the following can be calculated, by putting dead load = 0 (as we are considering live load only) and remembering that the negative moment in the unloaded span is the same as that at the support of the adjacent loaded span less the moment in column (see Fig. 80).

We may write down the results as follows (notation of Fig. 80):

$$M_4 = -\frac{Wl}{12}\left(\frac{KC + 8B}{KC + 6B}\right) \quad . \quad . \quad . \quad . \quad . \quad (4)$$

$$= -\frac{Wl}{12} \cdot F_{10}$$

where $$F_{10} = \frac{KC + 8B}{KC + 6B}$$

$$M_2 = -\frac{Wl}{12} \cdot \frac{2B}{KC + 6B} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

$$= -\frac{Wl}{12} \cdot F_{11}$$

where $$F_{11} = \frac{2B}{KC + 6B}$$

$$M_3 = -\frac{Wl}{12}\left(\frac{KC + 2B}{KC + 6B}\right) \quad . \quad . \quad . \quad . \quad . \quad (6)$$

$$= -\frac{Wl}{12} \cdot F_{12}$$

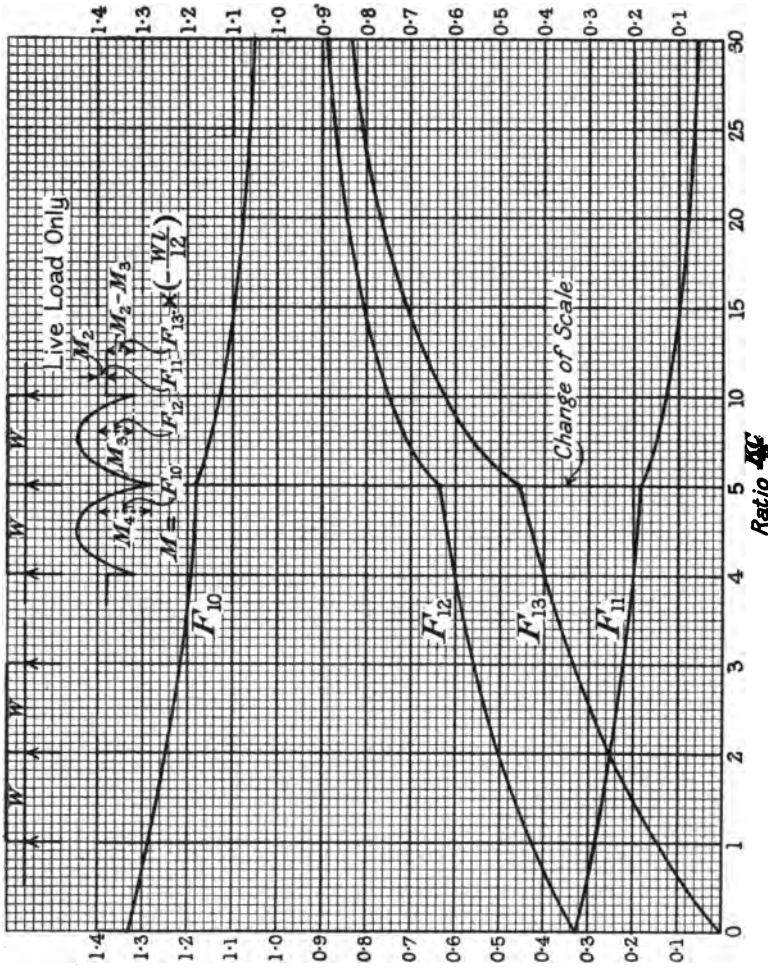



Fig. 80. Factors $F_{10}$, $F_{11}$, $F_{12}$ and $F_{13}$ of Negative Moments in Beams and Columns. Unequal Loadin g

where $$F_{12} = \frac{KC + 2B}{KC + 6B}$$

$M$ in column $= M_2 - M_3$

$$= + \frac{Wl}{12}\left(\frac{KC \cdot}{KC + 6B}\right) \quad . \quad . \quad . \quad . \quad . \quad (7)$$

$$= \frac{Wl}{12} \cdot F_{13}.$$

These factors are evaluated in the following table, from which Fig. 80 was prepared.

TABLE IV. Adjacent bays loaded.

| $\frac{KC}{B}$ | $F_{10}$ | $F_{11}$ | $F_{12}$ | $F_{13}$ |
|---|---|---|---|---|
| 0 | 1·33 | ·333 | ·333 | 0 |
| ·5 | 1·31 | ·308 | ·385 | ·077 |
| 1·0 | 1·285 | ·286 | ·425 | ·143 |
| 2 | 1·25 | ·25 | ·5 | ·25 |
| 5 | 1·18 | ·182 | ·637 | ·455 |
| 10 | 1·13 | ·125 | ·75 | ·625 |
| 20 | 1·08 | ·077 | ·846 | ·77 |
| 30 | 1·05 | ·055 | ·889 | ·833 |
| ∞ | 1 | 0 | 1·0 | 1·0 |

Note that when the columns are very stiff $\left(\frac{KC}{B} = \infty\right)$, the moment at the support then reduces from $\frac{Wl}{9}$ to $\frac{Wl}{12}$. Now all these quantities were necessary for drawing the complete moment diagram. But as all the values of $M_3$, $M_2$, and the column moment give lesser results than the values obtained from Condition ($a$) (alternate bays loaded) and as we can draw the cusp near enough for most practical work by eye when we know $M_4 = F_{10} \cdot \frac{Wl}{12}$ we only need for designing the beams $F_8$, $F_9$ and $F_{10}$. To avoid confusion these are given alone in Fig. 81.

From these figures the curves in Fig. 82, right-hand side, have been drawn and show very clearly the effect of stiff columns.

*End spans.*

To draw the diagrams we need to know the values of $M_1$, $M_2$, $M_3$, $M_4$ and $M_5$ (see diagrams on top of Fig. 82). Of these $M_1$ is

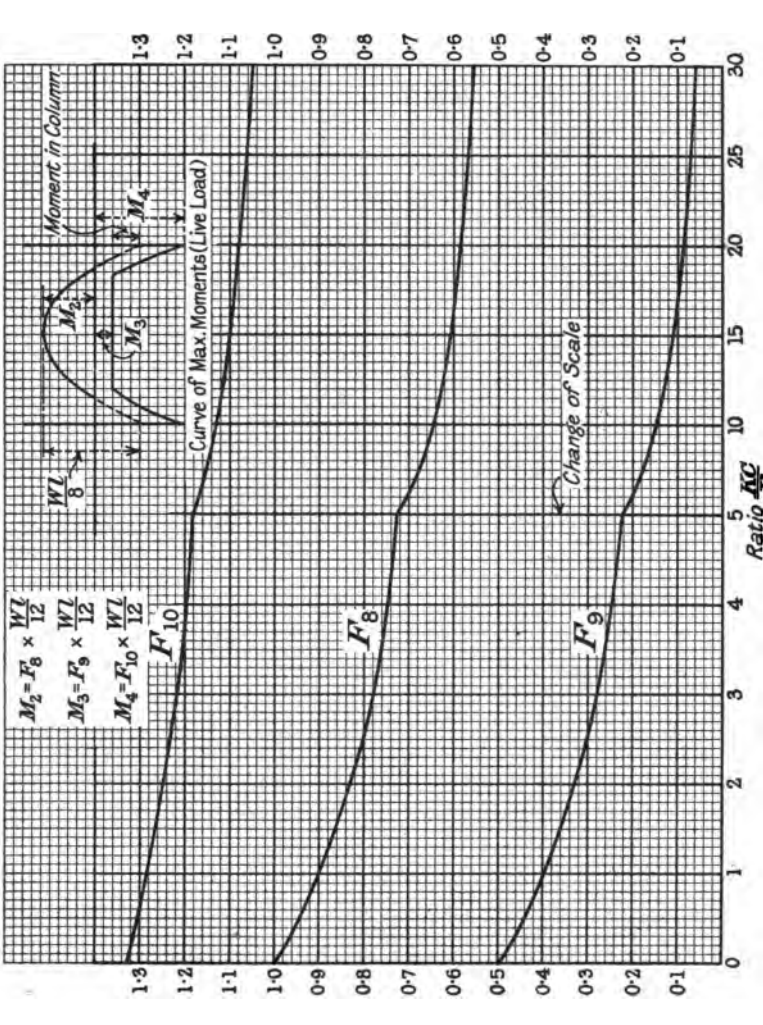



Fig. 81. Factors $F_8$, $F_9$, $F_{10}$ for a Maximum Live Load. Moment Curves Interior Span.

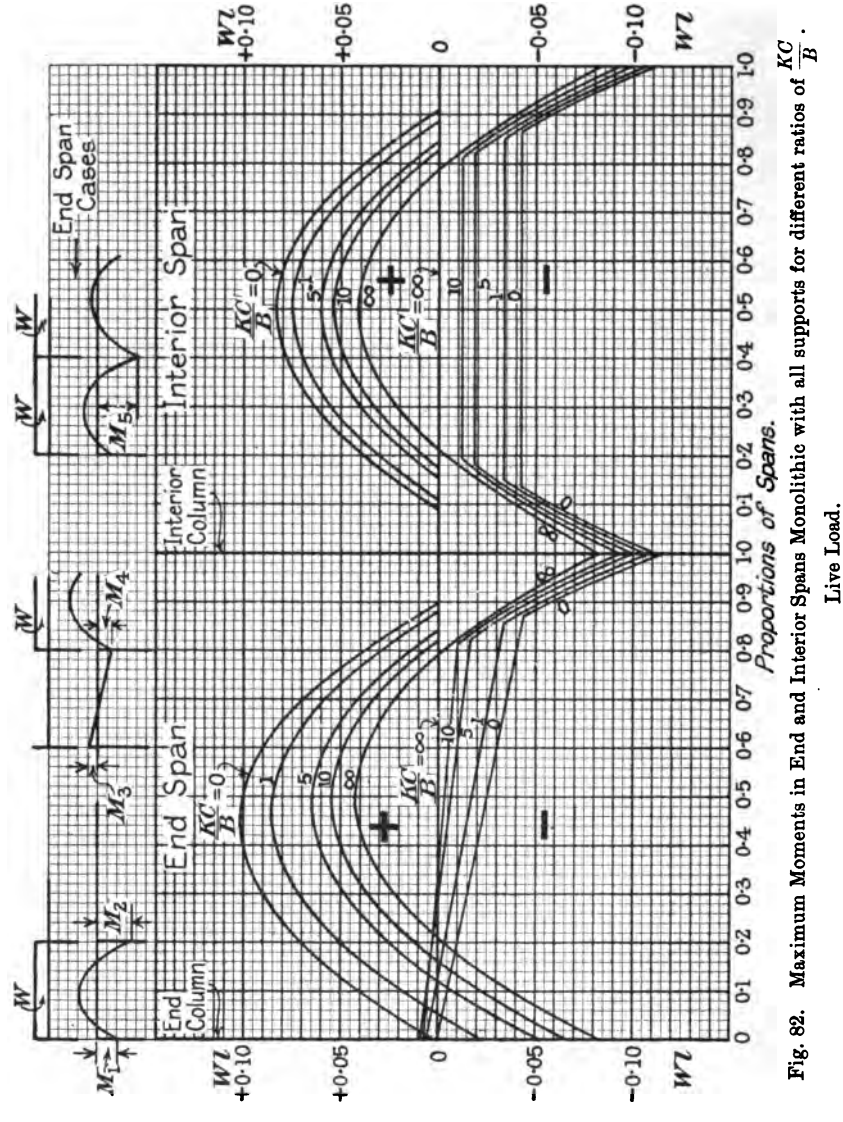



Fig. 82. Maximum Moments in End and Interior Spans Monolithic with all supports for different ratios of $\frac{KC}{B}$.
Live Load.

given by $\frac{Wl}{12} \times F_{15}$ (live load) (see p. 138 and Figs. 64 and 65); $M_3$ is given by $\frac{Wl}{12} \times F_{18}$ (see Fig. 76). Values for the others are as follows:

| $\frac{KC}{B}$ | $M_2$ | $M_4$ | $M_5$ |
|---|---|---|---|
| 0 | $\cdot 05Wl$ | $\cdot 05Wl$ | $\cdot 116Wl$ |
| 1 | $\cdot 057$ | $\cdot 04$ | $\cdot 111$ |
| 5 | $\cdot 07$ | $\cdot 022$ | $\cdot 101$ | 
| 10 | $\cdot 074$ | $\cdot 0143$ | $\cdot 096$ |
| $\infty$ | $\cdot 08\dot{3}$ | 0 | $\cdot 08\dot{3}$ |

*Note.* $M_5$ is practically the same as $F_{10} \times \left(-\frac{Wl}{12}\right)$.

From these figures, the left-hand curves in Fig. 82 have been drawn, and are most instructive.

*Practical application.*

Select the appropriate curves in Figs. 82 and 78 (corresponding to the value of $\frac{KC}{B}$ in the example) or draw them from the information given.

For any beam section read off positive and negative moment at that section from Fig. 82, apply this to the live load, and add to each of these values the moment from Fig. 78 applied to the dead load, having due regard to signs.

This gives the worst positive and negative moment at the section.

*Other conditions of loading.*

As before, with point loads at midspan, two point loads at third points, etc., the factors for moments in beams at supports, or in the columns, still apply but need multiplying by the appropriate *fixing moments*, some of which are as below:

uniform load $-\frac{Wl}{12}$,

point load at midspan $-\frac{Wl}{8}$,

2 point loads at third points $-\frac{Wl}{9}$.

Figs. 83–89 show the corresponding moment curves, taking column stiffness into account for beams of one, two and many spans, with central point load and two point loads at third points. From what has gone before, their use, derivation and method of application should need no further comment.

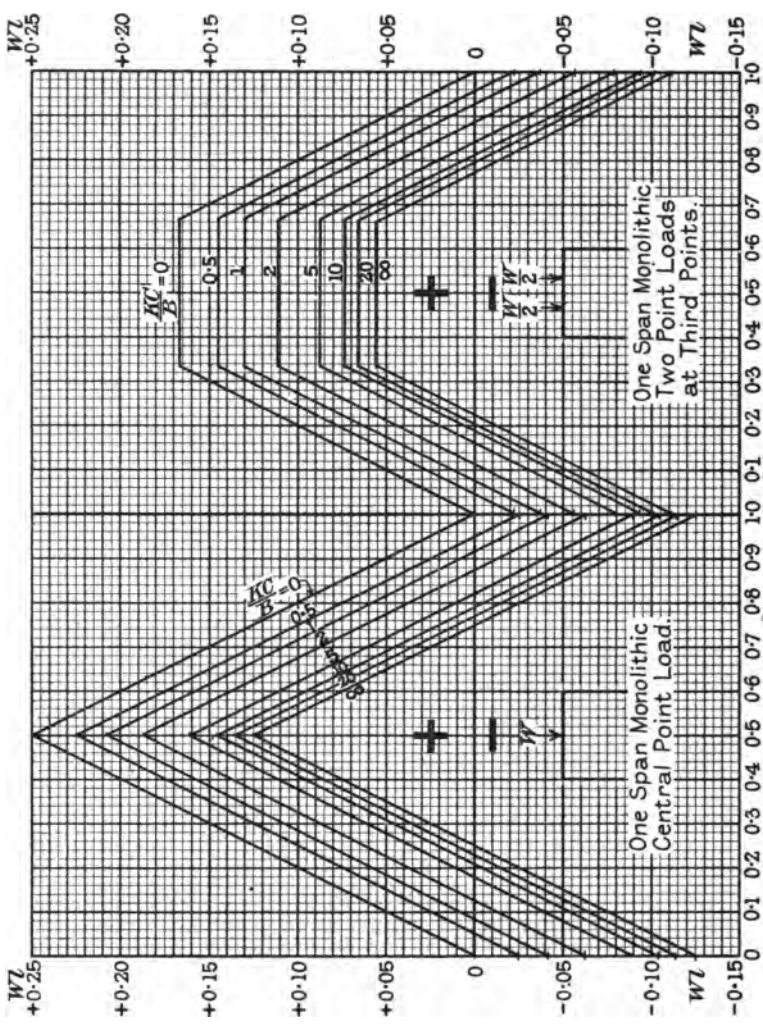



Fig. 83. Moment Curves. One Span Monolithic. Central Point Load and Two Point Loads at Third Points.

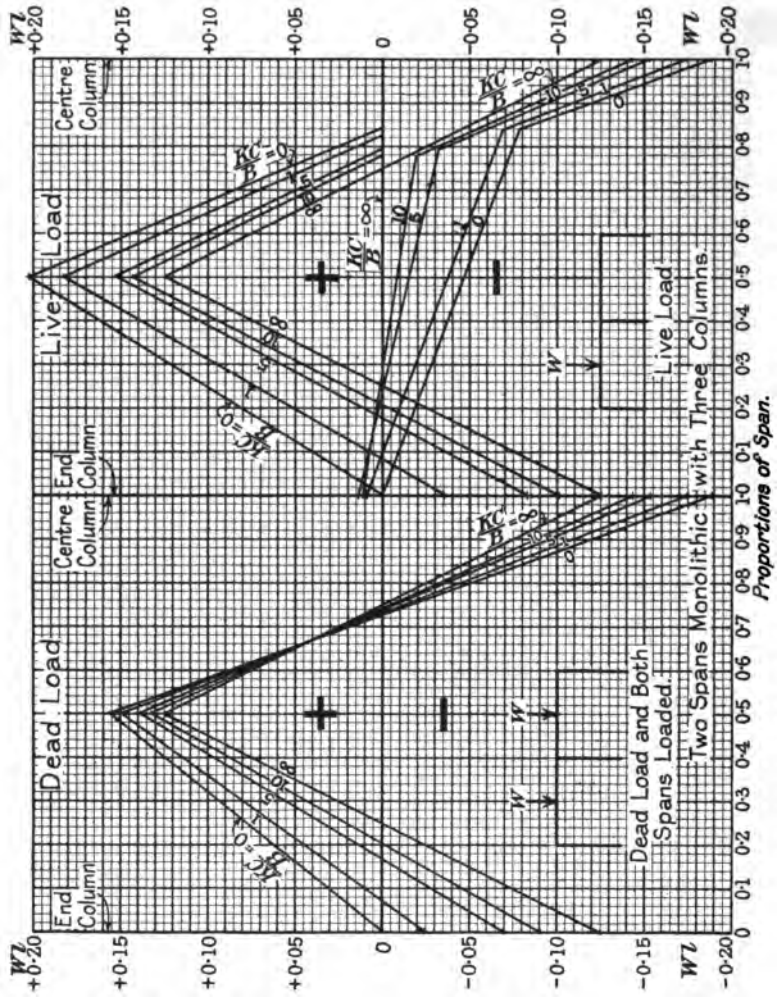



Fig. 84. Moment Curves. Two Spans Monolithic with Three Columns. Central Point Load. Dead Load and Live Load.

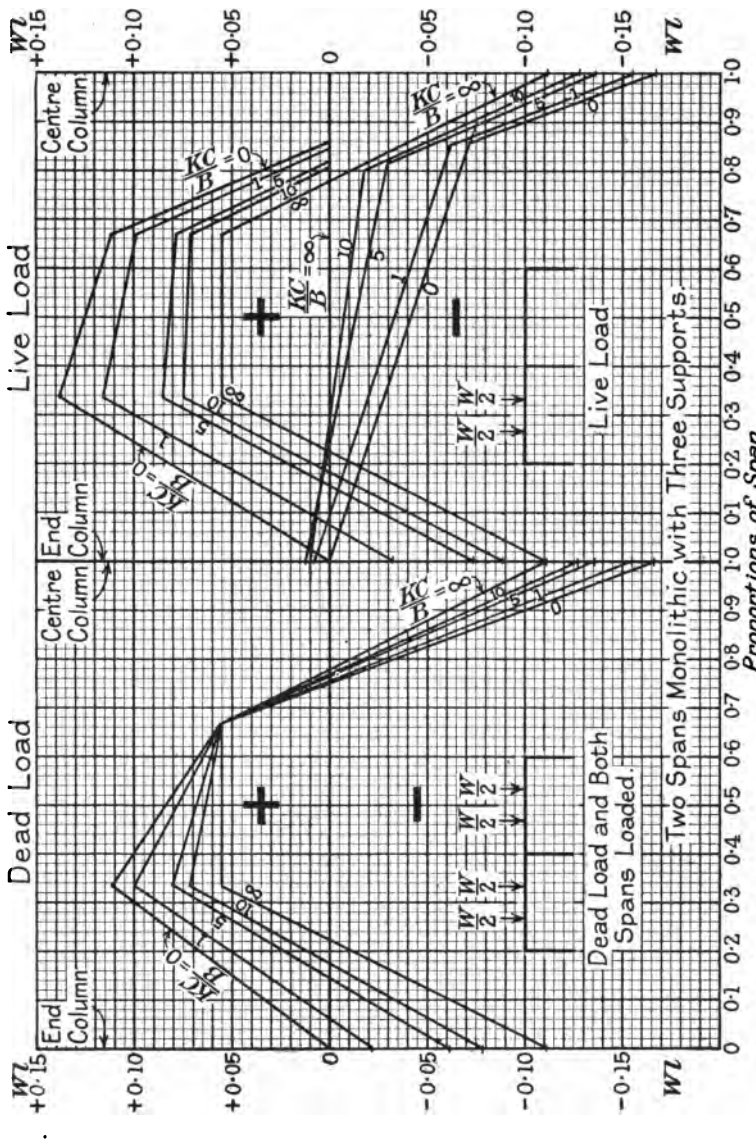



Fig. 85. Moment Curves. Two Spans Monolithic with Three Columns. Two Point Loads at Third Points. Dead Load and Live Load.

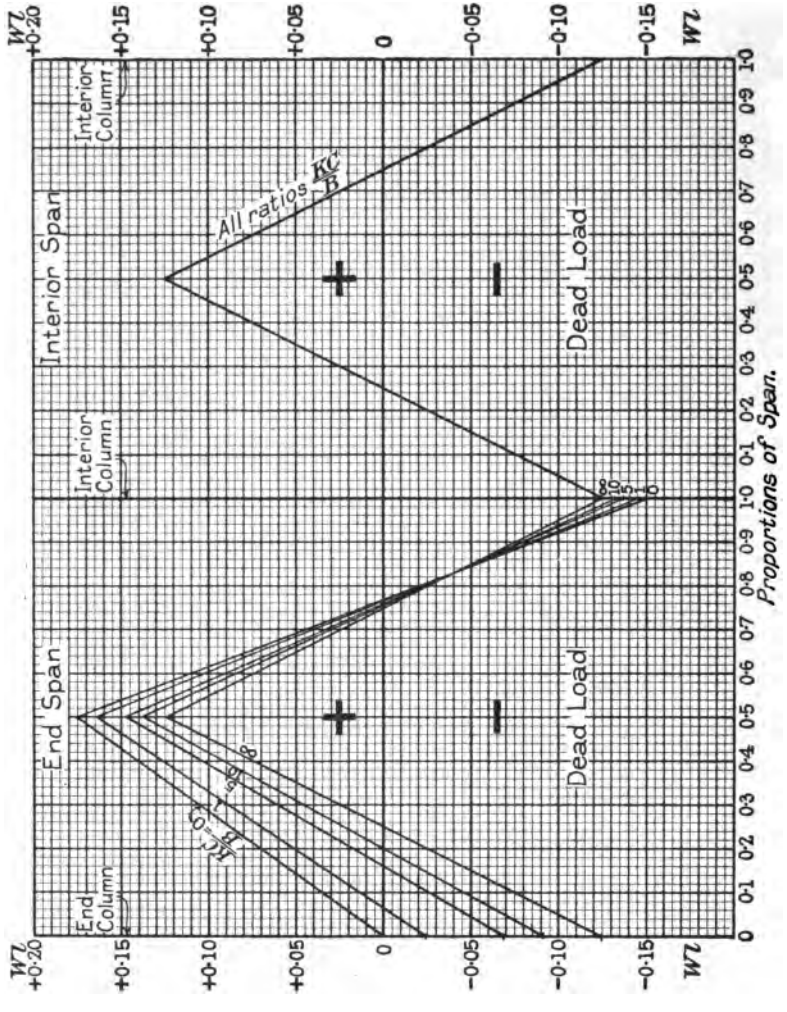



Fig. 86. Moment Curves. Many Spans all Monolithic with all Columns. Point Loads at Midspan. Dead Load or all Spans Loaded.

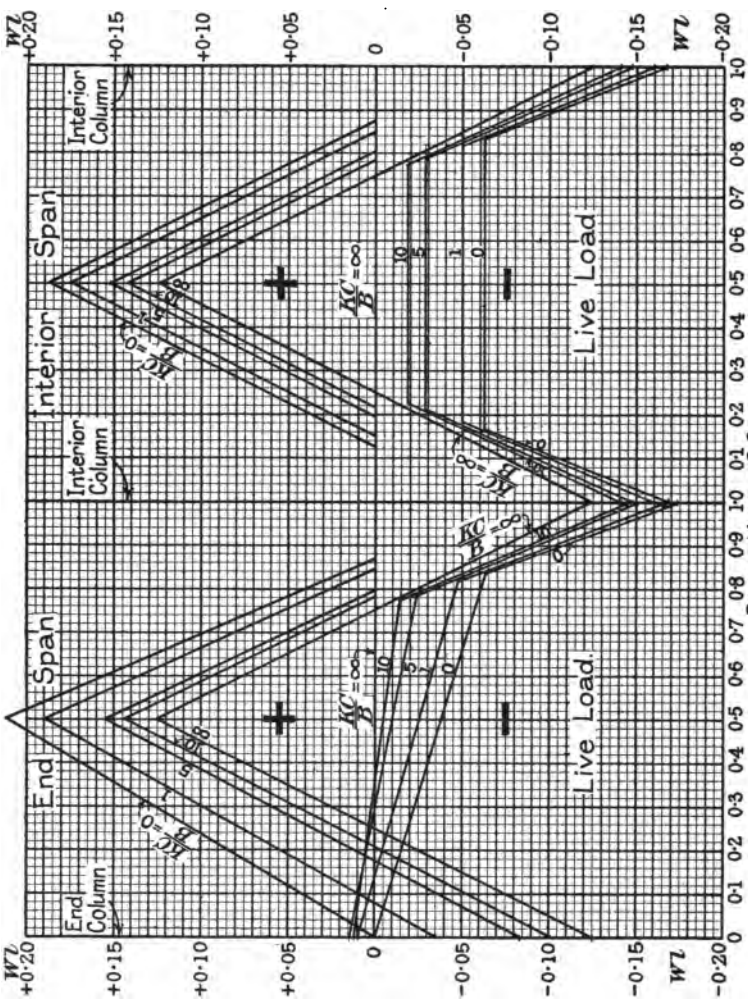



Fig. 87. Moment Curves. Many Spans all Monolithic with all Columns. Point Loads at Midspan. Live Load.

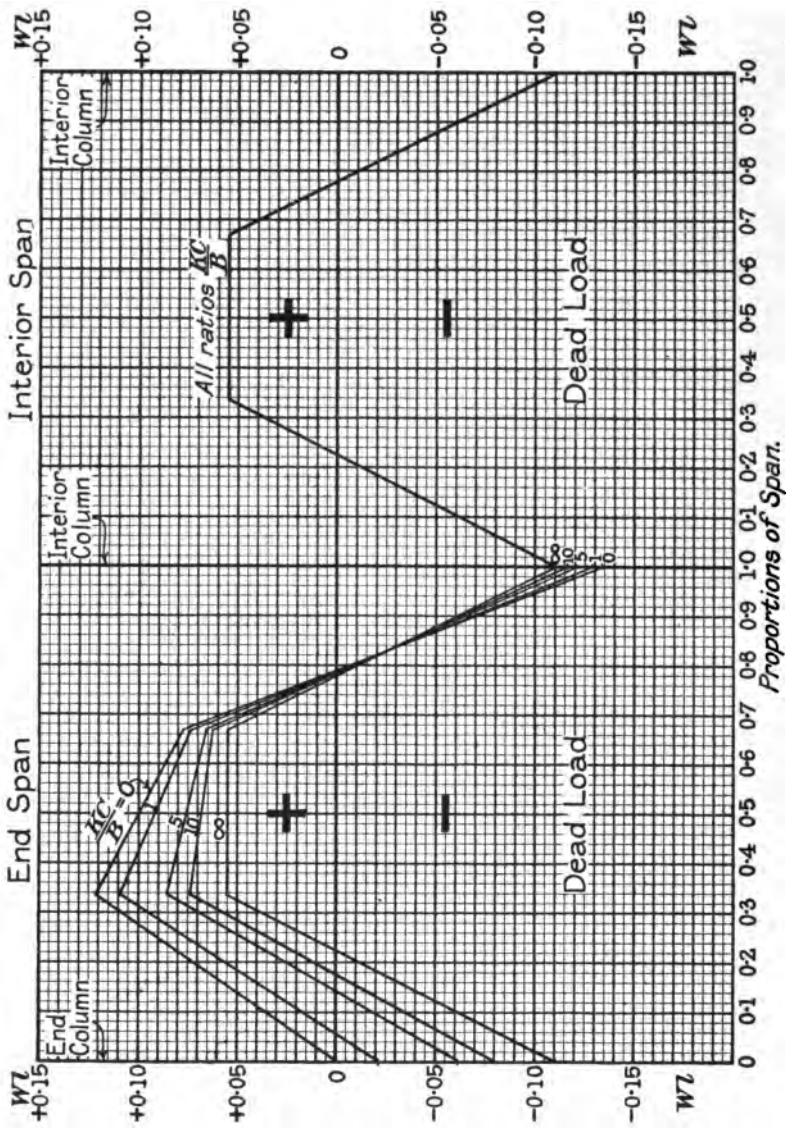



Fig. 88. Moment Curves. Many Spans all Monolithic with all Columns. Two Point Loads at Third Points. Dead Load or all Spans Loaded.

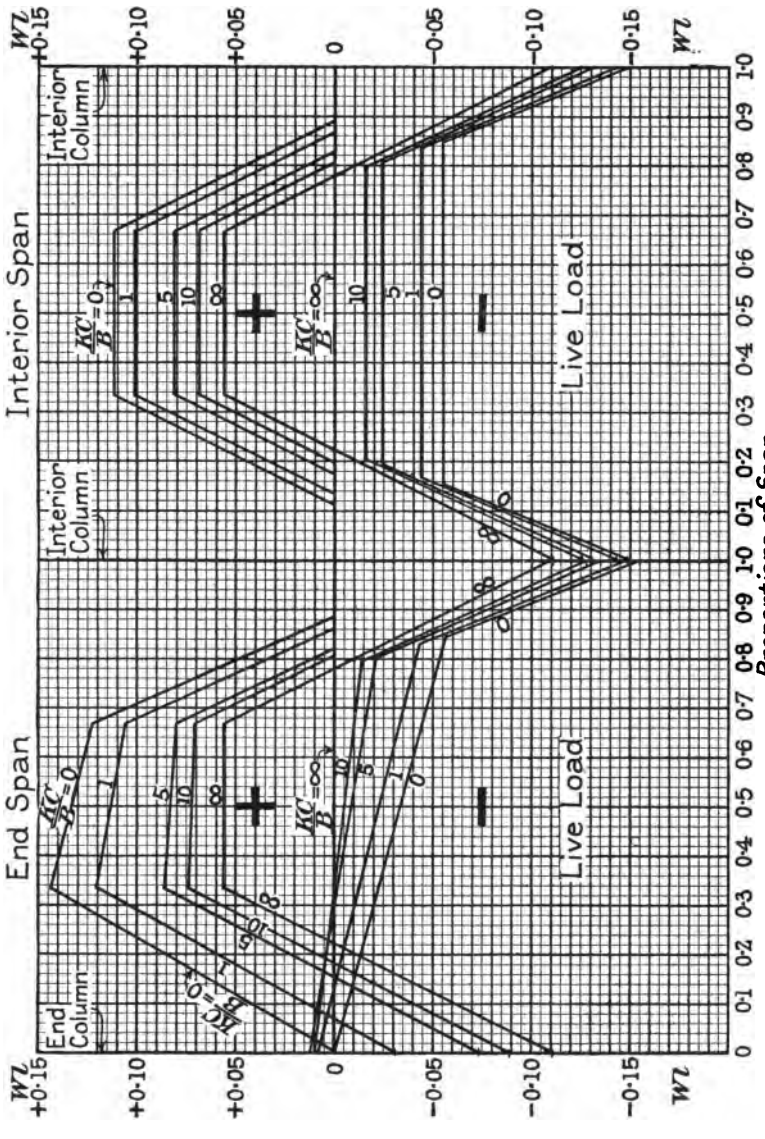



Fig. 89. Moment Curves. Many Spans all Monolithic with all Columns. Two Point Loads at Third Points. Live load.

# APPENDIX III

## REINFORCED CONCRETE REGULATIONS

### LONDON COUNTY COUNCIL (GENERAL POWERS) ACT, 1909

### REINFORCED CONCRETE CONSTRUCTION.

*Sub-headings, marginal notes and italic cross references do not form part of the regulations.*

REGULATIONS with respect to the construction of buildings wholly or partly of reinforced concrete and with respect to the use and composition of reinforced concrete in such construction.

WHEREAS under the provisions of section 23 of the London County Council (General Powers) Act, 1909, the London County Council (therein and hereinafter referred to as "the Council") have power to make regulations with respect to the construction of buildings wholly or partly of reinforced concrete and with respect to the use and composition of reinforced concrete in such construction.

NOW THEREFORE in pursuance of the powers conferred upon the Council by the said section the Council do hereby make the following regulations—

## Part I

### GENERAL.

1. The term "reinforced concrete" as occurring in these regulations shall mean concrete which is reinforced by metal so combined therewith that the metal will Reinforced concrete defined.

(*a*) be sufficient to take up all the tensile stresses;

(*b*) assist in the resistance to shear;

(*c*) assist in the resistance to compression where necessary.

Scope.

2. These regulations shall apply only to the construction of buildings of reinforced concrete in which the loads and stresses are transmitted through each storey to the foundations by a skeleton framework of reinforced concrete or partly by a skeleton framework of reinforced concrete and partly by a party wall or party walls.

Skeleton framework.

3. The skeleton framework of reinforced concrete of a building together with the party wall or party walls (if any) upon which such framework bears shall be capable of safely and independently sustaining the whole dead load and the superimposed load bearing upon such framework and party wall or party walls, calculated in accordance with the data hereinafter contained.

Floors, stairs, roofs, etc.

4. All floors, stairs, landings, and other portions of a building carried by a reinforced concrete framework shall be constructed throughout of incombustible materials, and be carried upon supports of incombustible materials; nevertheless, all internal stairs and landings may be constructed of fire-resisting materials, and nothing in this regulation shall prevent the use of wood framing, boarding and battens in the construction of roofs.

Electrical currents.

5. No part of the reinforcing metal in a building shall be used for conducting electrical currents.

Notices and plans.

6. In the case of the erection of a new building of reinforced concrete or the making of any addition or alteration or the carrying out of other work under the provisions of these regulations, the like notice shall be served upon the district surveyor as is required to be served by Part XIII of the London Building Act, 1894, and such notice shall be accompanied (*a*) in the case of a new building by plans and sections of sufficient detail to show the construction thereof, together with a copy of the calculations of the loads and stresses to be provided for and particulars of the materials to be used; and should such plans, sections, calculations or particulars as originally furnished be not in sufficient detail, the person depositing the same shall furnish the district surveyor with such further plans, sections, calculations or particulars as may be reasonably

necessary for the aforesaid purpose, and (*b*) in the case of an alteration or addition or other work as aforesaid by such plans, sections, calculations and particulars as may be reasonably necessary.

## Part II

### Data to be Used for the Purposes of these Regulations.

*Floor and roof loads.*

Dead load defined.

7. The dead load of a building shall consist of the actual weight of walls, floors, stairs, landings, roofs and all other permanent construction comprised in such building.

Superimposed load defined.

8. The superimposed load in respect of a building shall consist of all loads other than the dead load.

Loads.

9. In calculating the loads on foundations, pillars, piers, walls, framework, beams and other constructions carrying loads in buildings, the superimposed load shall be estimated as equivalent to the dead load set forth in the regulations following, and numbered *ten* to *twenty-four*, both inclusive.

Floor loads.

10. For a floor the estimate shall be made in accordance with the table immediately following—

| For a floor intended to be used for the purpose of | Equivalent dead load in pounds per square foot |
|---|---|
| Domestic buildings not hereinafter specified<br>Other similar purposes ... ... ... | 70 |
| Asylum wards ... ... ... ...<br>Common lodging-house bedrooms ...<br>Hospital wards ... ... ... ...<br>Hotel bedrooms ... ... ... ...<br>Workhouse wards ... ... ... ...<br>Other similar purposes ... ... ... | 84 |
| Counting houses ... ... ... ...<br>Offices ... ... ... ... ...<br>Other similar purposes ... ... ... | 100 |

| For a floor intended to be used for the purpose of | Equivalent dead load in pounds per square foot |
|---|---|
| Art galleries ... ... ... ... ...<br>Chapels ... ... ... ... ...<br>Churches ... ... ... ... ...<br>Classrooms in school buildings, not being dwelling houses so used ... ... ...<br>Lecture rooms ... ... ... ...<br>Meeting rooms ... ... ... ...<br>Music halls ... ... ... ... ...<br>Public assembly ... ... ... ...<br>Public concert rooms ... ... ...<br>Public library reading rooms ... ...<br>Retail shops ... ... ... ... ...<br>Theatres ... ... ... ... ...<br>Workshops ... ... ... ... ...<br>Other similar purposes ... ... ... | 112 |
| Ball rooms ... ... ... ... ...<br>Drill rooms ... ... ... ... ...<br>Other similar purposes ... ... ...<br>Similar floors subject to vibration ... | 150 |
| Book stores at libraries ... ... ...<br>Museums ... ... ... ... ...<br>For every floor in a building of the warehouse class except such floors as are intended to be used for any of the purposes aforesaid—not less than ... ... | 224 |

Loads on stairs and landings.

11. For stairs and landings the superimposed load shall be estimated as equivalent to a distributed dead load of at least 120 pounds per square foot, but each step shall be capable of safely supporting a concentrated load at any point of not less than 300 pounds.

Loads on roofs.

12. For a roof the plane of which inclines upwards at a greater angle than twenty degrees with the horizontal, the superimposed load, which shall for this purpose be deemed to include wind pressure and weight of snow and ice, shall be estimated at twenty-eight pounds per square foot of sloping surface, normal to such sloping surface, on either side of such roof.

13. For all other roofs the superimposed load shall be estimated at fifty-six pounds per square foot measured on a horizontal plane.

Greater loads.

14. If the superimposed load on any floor, landing, stairs or roof is to exceed that hereinbefore specified for

such floor, landing, stairs or roof, such greater load shall be provided for with an equivalent margin of safety.

Loads not otherwise specified.

15. In the case of any floor or roof intended to be used for a purpose for which a superimposed load is not specified in these regulations, the superimposed load to be carried on such floor or roof shall be provided for with an equivalent margin of safety.

Rolling loads.

16. In cases where a rolling load actuated by mechanical power is to be provided for such rolling load shall be taken as equivalent to a static load 50 per cent. in excess of the actual rolling load. The positive and negative bending moment at every cross section due to every position of the rolling load shall be properly provided for.

Partitions.

17. Partitions and other structures superimposed on floors and roofs may be included in the superimposed load, provided the weight of the partition or other structure per square foot of base does not exceed the permissible load per square foot of floor or roof area. Partitions and other structures of greater weight shall be specially provided for.

Angle of dispersion.

18. For calculating the resistance moment the angle of dispersion of a point load through hard filling and concrete shall not be taken at more than forty-five degrees from the vertical.

Loads.

19. For the purpose of calculating the total load to be carried on foundations, pillars and walls in buildings of more than two storeys in height, the superimposed loads for the roof and for the floor of the topmost storey shall be calculated in full, but for the floors of the lower storeys a reduction of the superimposed loads shall be allowed as follows:

For the floor of the storey next below the topmost storey a reduction of five per cent. of the superimposed load on the floor of that storey, as calculated in pursuance of the foregoing regulations, shall be allowed; for the floor of the next storey ten per cent. of the superimposed load on the floor of that storey similarly calculated; and so on

by increments of five per cent. per storey till the reduction amounts to 50 per cent. It shall be taken at 50 per cent. per storey for all floors below.

20. No such reduction as aforesaid shall be allowed in the case of a building of the warehouse class.

*Wind pressure.*

Wind pressure.

21. All buildings shall be so designed as to resist safely a horizontal pressure equivalent to a static pressure of not less than twenty pounds per square foot of the whole projected surface normal to the direction of every wind.

(*For pressure on panels of external walls see regulation* 130.)

22. All structures or attachments whatsoever in connection with a building, including towers or other parts which extend above the roof, flat, or gutter adjoining thereto, shall be so designed as to resist safely a horizontal pressure equivalent to a static pressure of not less than forty pounds per square foot of the whole projected surface normal to the direction of every wind.

*Working load.*

Working load.

23. The working load shall include the superimposed load, and the dead load inclusive of the weight of the beams, slabs, pillars, or other members, and any plastering, tiles, mosaic, granitic or other similar finishing materials.

*Weight.*

Weight.

24. For the purposes of calculation, the average weight of reinforced concrete, together with any finishing materials as aforementioned, shall be taken at not less than 144 pounds per cubic foot measured over finished surfaces.

*Ratio of span to depth of a beam.*

Ratio of span to depth.

25. For the purpose of determining the ratio of span to depth of a beam, the effective depth of the beam shall be taken.

26. The effective depth shall be measured from the compressed edge of the constructional concrete to the common centre of gravity of the tensile reinforcement.

27. The ratio of the span of a beam to its effective depth shall not exceed the lesser of the two following ratios—

$$20 \times \frac{\text{tensile stress in regulation 43}}{\text{actual maximum tensile stress}}$$

or

$$20 \times \frac{\text{compressive stress in regulation 42}}{\text{actual maximum compressive stress}}$$

28. The length of a cantilever shall not exceed five times the effective depth at the bearing.

*Bending moments.*

Effective span.

29. For the purpose of ascertaining the bending moments on a beam or on a slab the effective span, and the whole load on the effective span shall be taken into account in the calculations.

30. In the case of non-continuous beams or slabs, the effective span shall be taken as the distance between the main vertical sides of the piers, pillars, or walls, plus the effective depth of the beam or slab at the supports, or the span between the centres of the necessary bearing surfaces, whichever may be the lesser.

31. In the case of continuous beams or slabs the effective span shall be taken as the clear span plus the effective depth of the beam or slab, or the length between the centres of the supports, whichever may be the lesser.

Fixed ends defined.

32. A beam or slab shall be deemed to have fixed ends when its ends are sufficiently secured to other parts of the construction having such rigidity as will maintain the neutral planes of the beam at the ends in their original positions and directions under all variations in the incidence and intensity of loading.

Maximum moments.

33. The bending moments to be provided for at every cross-section of a beam or of a slab shall be the maximum

positive and negative moments at such cross-section. Such maximum bending moments shall be calculated in a manner consistent with regulations 34, 35, 36, 37, and in accordance with regulations 38 and 39.

### BEAMS WITH ENDS FREE OR FIXED.

*Notation for bending moments.*

34. $B$ = *bending* moment.
$b$ = *breadth.*
$l$ = *length* of effective span.
$W$ = total *weight* or *working* load.

| Load | Supports | Value of $B$ to be provided for |
|---|---|---|
| Concentrated at free end of cantilever | Cantilever ... | $B = -Wl$ at fixed end |
| Uniformly distributed... | Cantilever ... | $B = -\frac{Wl}{2}$ at fixed end |
| Concentrated at centre of beam | Both ends freely supported | $B = \frac{Wl}{4}$ at centre |
| Uniformly distributed... | Both ends freely supported | $B = \frac{Wl}{8}$ at centre |
| Concentrated at centre | Both ends fixed | $B = \frac{Wl}{8}$ at centre |
| Concentrated at centre | Both ends fixed | $B = -\frac{Wl}{8}$ at ends |
| Uniformly distributed... | Both ends fixed | $B = -\frac{Wl}{12}$ at ends |
| Uniformly distributed... | Both ends fixed | $B$ at centre shall not be taken at less than the numerical value of $B$ at the ends |
| Uniformly distributed... | One end fixed... One end freely supported | $B = -\frac{Wl}{8}$ at fixed end |
| Uniformly distributed... | One end fixed... One end freely supported | $B = \frac{Wl}{14}$ at $\frac{3}{8}$ of span from the free end |

## CONTINUOUS BEAMS.

*Notation for bending moments.*

Continuous beams.

35. $B$ = *bending* moment.
$l$ = *length* of effective span measured between the centres of supports. (*See regulation* 31.)
$W$ = total *weight* or *working* load. (Superimposed and dead loads.) (*See regulation* 23.)
$W_d$ = total *dead weight* per span.
$W_s$ = *superimposed weight* or load per span (uniformly distributed).

The total *bending* moment ($B$) to be provided for at every cross-section shall be the algebraic sum of the bending moments due to the superimposed and dead loads respectively.

Maximum bending moments due to variations in the incidence of distributed loads over approximately equal spans.

| Near middle of end span | At support next to end support | At the middle of interior spans | At other interior supports |
|---|---|---|---|
| $+\frac{W_s l}{10}$ $+\frac{W_d l}{12}$ | $-\frac{W_s l}{9}$ $-\frac{W_d l}{10}$ | $+\frac{W_s l}{12}$ $+\frac{W_d l}{24}$ | $-\frac{W_s l}{9}$ $-\frac{W_d l}{12}$ |
| or approximately $+\frac{Wl}{10}$ | or approximately $-\frac{Wl}{10}$ | or approximately $+\frac{Wl}{12}$ | or approximately $-\frac{Wl}{10}$ |

36. Notwithstanding anything in regulation 35, beams may be designed for the exact positive and negative bending moments which will occur at every cross-section whether all the spans be loaded or alternate or any of the spans be unloaded.

Bending moments on slabs.

37. (*a*) Bending moments on slabs supported by, fixed at, or continuous over four edges, shall be calculated as for beams.

(*b*) When reinforced with mesh reinforcement or reinforced in two directions at right angles to each other,

the load assumed to be carried in each direction may be apportioned as follows—

Proportion of load assumed to be carried in the direction of the *breadth* of the slab—

$$= \frac{1}{1 + \left(\frac{b}{l}\right)^4} \times \text{total load on slab.}$$

Proportion of load assumed to be carried in the direction of the *length* of the slab—

$$= \frac{1}{1 + \left(\frac{l}{b}\right)^4} \times \text{total load on slab.}$$

Provided that when the length of a slab exceeds twice its breadth the whole load shall be assumed to be carried in the direction of the breadth of such slab.

Moments over supports.

38. The maximum area of steel required to take the negative bending moment at any support shall be carried without reduction across the centre of that support. (*Also see regulation* 35.)

Variations of loading.

39. To allow for variations in the incidence of the loading on contiguous spans of continuous beams or slabs, the bending moments to be provided for at every cross-section shall be the maximum positive and negative moments at such cross-section, whether all the spans be loaded or alternate or any of the spans be unloaded.

Conditions of loading not otherwise specified.

40. For every condition of loading not specified in these regulations, the bending moment for beams or slabs shall be calculated so as to give an equivalent margin of safety.

Reinforcement at points of contraflexure.

41. Reinforcement shall be carried beyond the points of contraflexure under any condition of loading, by a length at least equal to half the effective depth of the beam.

*Working stresses.*

42. (*a*) Except as further provided for in pillars, the permissible working stresses in concrete shall not exceed the following— Stresses in concrete. Also see regulations 145, 148, 156, 157, 158, 159, 162.

| Stresses in concrete | Proportion by volume | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Cement | Sand | Coarse material | Cement | Sand | Coarse material | Cement | Sand | Coarse material | Cement | Sand | Coarse material |
| | 1 | 2 | 4 | 1·2 | 2 | 4 | 1·5 | 2 | 4 | 2 | 2 | 4 |
| | $V=6$ | | | $V=5$ | | | $V=4$ | | | $V=3$ | | |
| | Pounds per sq. in. | | | Pounds per sq. in. | | | Pounds per sq. in. | | | Pounds per sq. in. | | |
| Direct compressive stress | 600 | | | 650 | | | 700 | | | 750 | | |
| Extreme flexural compressive stress in beams | 600 | | | 650 | | | 700 | | | 750 | | |
| Shearing stress ... ... | 60 | | | 65 | | | 70 | | | 75 | | |
| Grip or adhesion between concrete and steel bars hooked at both ends | 100 | | | 100 | | | 100 | | | 100 | | |
| Grip or adhesion between concrete and steel bars otherwise effectively anchored at the ends | 60 | | | 60 | | | 60 | | | 60 | | |
| Tensile stress ... ... | nil | | | nil | | | nil | | | nil | | |

See regulation 100.

(*b*) The values of the permissible *compressive* stress in pounds per square inch for intermediate proportions may be estimated from the following equation—

$$c = 900 - 50\,V$$

where $V$ equals the *volume* of the sand plus that of the coarse material, per volume of cement, each measured separately and including the voids proper to each material.

If $V$ is less than 3, the stresses in the last column shall not be exceeded.

(*c*) The values of the shearing stress shall not exceed one-tenth of the direct compressive stress for any given value of $V$.

Stress in steel.

43. The permissible working stress in the steel shall not exceed the following—

| Stresses in mild steel complying with the British Standard Specification | Pounds per square inch |
|---|---|
| Compressive stress ... ... | $m$ times the stress in the concrete immediately surrounding the steel (the value of $m$ being obtained in accordance with regulation 54). See also regulations 61 and 62 |
| Tensile stress ... ... | 16,000 |

Eccentric loading.

44. Provision shall be made for the stresses due to eccentric or non-axial loading.

Combined stresses.

45. The term "combined stresses" when used in these regulations shall mean the resultant of all the stresses due to any cause or causes.

46. Neither the steel nor the concrete shall be subjected to combined stresses which exceed the permissible working stress.

Connections.

47. All connections in or between reinforced concrete members shall be designed and arranged so that the stresses which may come upon them shall be within the limits allowed by these regulations.

Hooks and Anchors.

48. (*a*) All tensile and shear reinforcement shall be hooked or otherwise effectively anchored at each end.

(*b*) Hooks at the ends of bars shall be of a ⊃ form, and shall have an inner diameter of at least four times the diameter of the bar; or when the hook fits over a main reinforcing bar the diameter of the hook may be equal to the diameter of such bar. The length of the straight part beyond the end of the curve shall be at least four times the diameter of the bent bar.

(*c*) When the bars are anchored at each end by means other than a hook complying with paragraph (*b*) of this regulation, the anchorage shall be bent up to a right angle from the centre line of the bar. The width across such anchorage shall be at least three times the normal diameter of the anchored bar. (*Also see regulation* 139.)

*Grip length.*

49. The grip length or adhesion length of a bar embedded in concrete shall be measured along the bar from the beginning of the curve of the hook or the nearer end of the other form of anchorage. Grip.

50. The grip or adhesion length shall be sufficient to keep the stresses within the limits set out in regulation 42, except for web reinforcement complying with regulations 48 (*b*) and 67.

51. For the purpose of ascertaining the necessary grip or adhesion length in the case of a bar having mechanical bond the perimeter of the bar may be measured over the transverse projections provided—

(*a*) That the transverse projections are not further apart (centre to centre) than twice the normal diameter of the bar;

(*b*) That the height of such projections above the normal surface of the bar shall be at least 1/10th of the diameter of the bar.

*Modular Ratios.*

52. The term "modular ratio" (*m*) means the ratio of the elastic modulus of steel to the elastic modulus of the concrete. Modular ratios.

53. The elastic modulus for steel in tension or compression shall be taken at thirty million pounds per square inch. (*See regulation* 80 (*c*).)

See regulations 83 and 114.

54. The modular ratio for steel and concrete shall be taken as follows—

| | |
|---|---|
| For beams ... ... | $m.c. = 9000$ or $m = \frac{9000}{c}$ |
| For pillars with the minimum of hooping or binding ... ... | $m.c. = 9000$ or $m = \frac{9000}{c}$ |
| For pillars with more than the minimum of hooping or binding ... | $m.i. = 9000$ or $m = \frac{9000}{i}$ |

Where $c$ = permissible *compressive* working stress given in regulation 42 and $i$ = *increased* stress given by regulation 115.

## Part III

### BEAMS.

Beam defined.

55. The term "beam" shall include any beam, girder, lintel, bressummer or cantilever, or any other similar member carrying transverse loads.

Diameter of bars.

56. The least diameter or thickness of the longitudinal bars in beams shall not be less than one-quarter of an inch.

57. All other reinforcements in beams shall be at least one-eighth of an inch in diameter or thickness.

Space between bars.

58. There shall be a distance of at least one inch horizontally and one half-inch vertically between the bars in beams except at joints or at points where the bars are in direct contact and transverse to one another.

59. The distance between the bars of the tensile reinforcement in a beam shall not be greater than six inches.

Wiring.

60. Wiring used in beams for the purpose of holding bars in position shall not be regarded as reinforcement.

*Compressive reinforcement.*

61. In cases where the compressive resistance of the concrete is taken into account the stress in the compressive reinforcement may be taken at an amount not exceeding $m$ times the stress in the concrete at the same distance from the neutral axis on condition that—

(*a*) The compressive reinforcement is anchored by bars extending at least through a depth equal to the arm of the resistance moment.

(*b*) The anchors are spaced not further apart (centre to centre) than a distance equal to the arm of the resistance moment, and not further apart (centre to centre) than 16 times the least diameter of the anchored bar. (*Also see regulations* 89 *and* 90.)

62*. In cases where the compressive resistance of the concrete is not taken into account the stress in the compressive reinforcement may be taken at 16,000 pounds per square inch on condition that—

(*a*) Sufficient steel is provided to take up the whole of the compression.

(*b*) The compressive reinforcement is anchored laterally and vertically by anchors not further apart (centre to centre) than 6 inches and not further apart (centre to centre) than 8 times the diameter of the anchored bar.

(*c*) The anchors shall be passed round or hooked over both the compressive and the tensile reinforcement.

Hooped core.

63. In cases where the concrete in compression is hooped at the ends of a beam, the stress within the hooped core, for the distance of one-quarter the span of the beam at each end, may be taken at the same value as for a core in a pillar similarly hooped. In this case the compressive stress in the concrete outside the hooping shall not be taken into account in estimating the compressive resistance.

* See p. 244.

*Shear or web reinforcement**.

Shear.

64. The vertical shear taken by the concrete only shall be calculated on the compressed area of the web or on the web area for a depth equal to the arm of the resistance moment of the beam. The intensity of the shearing stress shall not be greater than the values given in regulation 42.

65. Where the vertical shear is taken by the concrete only, in accordance with regulation 64, the ends of 50 per cent. of the bars of the tensile reinforcement shall be inclined across the neutral plane of the beam, and shall be carried through a depth equal to the arm of the resistance moment, or the whole of the bars shall be carried through to the ends of the beam. (*See regulations* 87 (*c*) *and* 88 (*c*).)

66. If the shearing stress at any cross-section, calculated on the concrete alone, is in excess of the permissible shearing stress, the whole shear shall be provided for by the tensile resistance of the shear or web reinforcement acting in conjunction with the compressive stresses in the web, but in no case shall the ratio $\frac{S}{b_r d}$ exceed three times the shearing stress given in regulation 42 (*a*), where

$b_r$ = mean *breadth* of the *rib* of a tee beam or the *breadth* of a *rectangular* beam;

$d$ = effective *depth* of the beam;

$S$ = total vertical *shearing* force at any cross-section.

(*Also see regulation* 84.)

67. Shear or web reinforcement shall—

(*a*) Be spaced according to the distribution and intensity of the shearing stresses, but the distance from centre to centre of the shear or web members at any part of the beam shall not exceed a length equal to the arm of the resistance moment.

(*b*) At least extend from the centre of the tensile reinforcement to the centre of pressure in the concrete under compression.

* See p. 244.

(*c*) Be passed under or round the tensile reinforcement or be otherwise secured thereto.

(*d*) Be hooked at both ends, in the same manner as for tensile reinforcement, or equally effectively anchored.

68. Tensile reinforcement which is inclined across the neutral plane of a beam, and which is carried through a depth equal to the arm of the resistance moment may be taken as shear or web reinforcement.

69. The shear or web reinforcement may be regarded as anchors under the regulations 61 and 62, provided it complies therewith.

*Splays and brackets at ends of beams and bracketed work.*

70. Where the end of a beam is splayed for the purpose of increasing the resistance moment, such splayed work shall not be calculated at a greater angle than 30 degrees from the horizontal. Splays.

71. In cases where bracketed work in reinforced concrete is provided in the structural design to carry loads, and to act as a cantilever, it shall be reinforced sufficiently to provide for the stresses due to such loads. Brackets.

SLABS.

72*. The effective depth of slabs shall be measured from the compressed edge of the constructional concrete to the centre of gravity of the tensile reinforcement. In the case of floor slabs the effective depth shall not be less than three inches. Effective depth.

73. The least diameter or thickness of independent reinforcing bars in slabs shall not be less than one-quarter of an inch. Diameter of bars.

74. The wires under tensile stress in connected mesh and similar reinforcement in slabs shall be at least one-tenth of an inch in diameter or thickness. Wires.

* NOTE.—Five inches is the least thickness for fire-resisting floors in the principal Acts.

Space between bars.

75. There shall be a distance of at least one inch between bars in slabs except at joints and at points where the bars are in direct contact and transverse to one another.

76. All meshed reinforcement shall be of such dimensions as will enable the coarse material in the concrete to pass easily through the meshes of such reinforcement.

77. The maximum distance between bars or strands of the tensile reinforcement in slabs shall not be greater than 12 inches, and not more than twice the effective depth of the slab.

Distributing bars.

78. In cases where independent reinforcing bars are provided in one direction only, distributing bars shall be provided on the top of the lower tensile bars at right angles thereto. Such distributing bars shall not be further apart than 18 inches, and shall have an aggregate cross-sectional area of at least 0·08 per cent. of the effective cross-sectional area of the slab; or the diameter of each of the distributing bars shall be at least 1/16th of the effective depth of the slab and the pitch of the distributing bars shall be not greater than four times the effective depth of the slab.

Wiring.

79. Wiring used in slabs for the purpose of holding bars in position shall not be regarded as reinforcement.

## Resistance Moments.

Resistance moments.

80. The resistance moment of reinforced concrete construction under transverse loads shall be determined by formulas based on the following assumptions—

(*a*) All tensile stresses shall be taken by the steel.

(*b*) The strain in any layer or fibre is directly proportional to the distance of that layer or fibre from the neutral axis.

(*c*) The tensile elastic modulus of steel shall be assumed to be equal to the compressive elastic modulus of that material. (*See regulation* 53.)

(*d*) The elastic moduli of the concrete remain constant within the limits of the working stress.

(*e*) The stress-strain graph is a straight line.

(*f*) The anchorage, together with the grip between the concrete and steel, is sufficient to make the two materials act together.

*Notation for beams and slabs.*

81. Notation.

$A$ = *area* of tensile reinforcement, in square inches.

$a$ = *arm* of the resistance moment, in inches.

$B$ = *bending* moment of the external loads and forces.

$b$ = *breadth* of rectangular beam in inches, or *breadth* of the flange of a tee beam in inches or *breadth* of slab in inches.

$c$ = permissible *compressive* working stress, at the extreme edge of the concrete in compression, in pounds per square inch.

$d_s$ = total *depth* of slab in inches.

$d$ = effective *depth* of the beam or slab in inches, i.e. the distance from the compressed edge of the constructional concrete to the common centre of gravity of the tensile reinforcement.

$E_c$ = *elastic* modulus of *concrete* in compression.

$E_s$ = *elastic* modulus of *steel* in tension or compression.

$l$ = *length* of the effective span of a beam or slab.

$m = \frac{E_s}{E_c}$ = *modular* ratio. (*See regulation* 54.)

$n$ = distance of the *neutral* axis from the compressed edge of the constructional concrete of the beam or slab, in inches.

$n_1 = \frac{n}{d}$ = *neutral* axis ratio, $\therefore n_1 d = n$.

$p$ = *percentage* of tensile reinforcement = $100r$.

$Q$ = *qualifier* in the equation $R = Qbd^2$.

$R$ = *resistance* moment generally.

$R_c$ = *resistance* moment of the internal stresses in the beam or slab in terms of the permissible *compressive* working stress.

$R_t$ = *resistance* moment of the internal stresses in the beam or slab in terms of the permissible *tensile* working stress.

$r$ = *ratio* of $A$ to $bd$, i.e. $r = \frac{A}{bd}$ and $A = rbd$.

$s_1$ = *slab* depth ratio $= \frac{d_s}{d}$.

$t$ = permissible *tensile* working stress, in tensile reinforcement, in pounds per square inch.

$t_1$ = ratio of the *tensile* stress in the steel to the compressive stress at the extreme edge of the concrete under flexure $= m\left(\frac{1}{n_1} - 1\right)$.

$W$ = total *weight*, or *working* load.

*Beams and slabs.*

Breadth of flange.

82. For the purpose of computing the resistance moment of a tee beam, the breadth of the flange shall not be taken at more than—

(*a*) One-fourth of the effective span of the tee beam;

(*b*) The distance between the centres of the ribs of the tee beams;

(*c*) Twelve times the thickness of the slab;

whichever is the least.

(*Also see regulations* 45 *and* 46 *as to combined stresses in beams, slabs, and other members.*)

83. In the case of ⌐ (ell) beams the breadth of the flange shall not be taken at more than four times the thickness of the slab, provided that where the moments due to the eccentricity of the compression are adequately provided for, the width may be increased, but not beyond one-half of that allowed for T (tee) beams.

Breadth of rib.

84. The minimum breadth of the rib of a T (tee) or ⌐ (ell) beam shall not be less than one-third the depth of the rib below the slab. (*Also see regulation* 66.)

Breadth of beam.

85. Rectangular beams shall be secured against buckling whenever the length of the beam exceeds 20 times

the least width thereof, or whenever the length of the beam exceeds 30 times the least width measured from outside to outside of the outermost longitudinal reinforcement.

Slab reinforcement.

86. When a part of a slab is taken as forming part of a tee beam, the reinforcement in the slab transverse to the beam must cross the full breadth of the portion of the slab forming the flange of the tee beam.

Neutral axis within the slab.

87. In the case of slabs, rectangular beams and tee beams reinforced in tension only when the neutral axis is within the slab, i.e. tee beams in which $r$ is less than

$$\frac{{s_1}^2}{2m\,(1 - s_1)}$$

(*a*) The position of the neutral axis shall be obtained from the equation

$$n_1 = \sqrt{(m^2r^2 + 2mr)} - mr$$

or

$$n_1 = mr\left[\sqrt{\left(1 + \frac{2}{mr}\right)} - 1\right]$$

or

$$n = [\sqrt{(m^2r^2 + 2mr)} - mr]\,d$$

(*b*) The mean compressive stress in the concrete shall be taken at $\frac{c}{2}$.

(*c*) The arm of the resistance moment shall be obtained from the equation

$$a = d - \frac{n}{3}$$

or

$$a = d\left(1 - \frac{n_1}{3}\right)$$

or approximately, for tee beams,

$$a = d - \frac{d_s}{3}$$

(*d*) The tensile resistance moment at every cross-section shall be at least equal to the bending moment at that section and shall be obtained from the equation

$$R_t = tA\left(d - \frac{n}{3}\right)$$

or $$R_t = tAd\left(1 - \frac{n_1}{3}\right)$$

or $$R_t = trbd^2\left(1 - \frac{n_1}{3}\right)$$

or $$R_t = Qbd^2 \text{ where } Q = tr\left(1 - \frac{n_1}{3}\right)$$

(*e*) The compressive resistance moment at every cross-section shall be at least equal to the bending moment at that section and shall be obtained from the equation

$$R_c = \frac{c}{2}bn\left(d - \frac{n}{3}\right)$$

or $$R_c = \frac{cbd^2}{2}n_1\left(1 - \frac{n_1}{3}\right)$$

or $$R_c = Qbd^2 \text{ where } Q = \frac{c}{2}n_1\left(1 - \frac{n_1}{3}\right)$$

Neutral axis intersecting the rib.

88. In the case of tee beams, reinforced in tension only when the neutral axis intersects the rib, i.e. tee beams in which $r$ is greater than

$$\frac{{s_1}^2}{2m(1 - s_1)}$$

(*a*) The position of the neutral axis shall be obtained from the equation—

$$n_1 = \frac{{s_1}^2 + 2mr}{2(s_1 + mr)}$$

(*b*) The mean compressive stress in the concrete shall not be taken at more than

$$c\left(1 - \frac{s_1}{2n_1}\right)$$

or $$\frac{cmr(2 - s_1)}{{s_1}^2 + 2mr}$$

(*c*) The arm of the resistance moment = $a$ where

$$a = d\left[1 - \frac{s_1}{3}\left(\frac{3n_1 - 2s_1}{2n_1 - s_1}\right)\right]$$

or $$a = d\left[\frac{{s_1}^3 + 4mr{s_1}^2 - 12mrs_1 + 12mr}{6mr(2 - s_1)}\right]$$

or approximately $$a = d - \frac{d_s}{2}$$

(*d*) The tensile resistance moment at every cross-section shall be at least equal to the bending moment at that section and shall be obtained from the equation

$$R_t = tAa,$$

or $$R_t = tbd^2\left[\frac{s_1^3 + 4mrs_1^2 - 12mrs_1 + 12mr}{6m\,(2 - s_1)}\right]$$

or $$R_t = Qbd^2$$

where $$Q = t\left[\frac{s_1^3 + 4mrs_1^2 - 12mrs_1 + 12mr}{6m\,(2 - s_1)}\right]$$

(*e*) The compressive resistance moment at every cross-section shall be at least equal to the bending moment at that section and shall be obtained from the equation

$$R_c = c\left(1 - \frac{s_1}{2n_1}\right)bd_sa,$$

or $$R_c = cbd_sd\left[\frac{s_1^3 + 4mrs_1^2 - 12mrs_1 + 12mr}{6\,(s_1^2 + 2mr)}\right]$$

or $$R_c = Qbd^2$$

where $$Q = cs_1\left[\frac{s_1^3 + 4mrs_1^2 - 12mrs_1 + 12mr}{6\,(s_1^2 + 2mr)}\right]$$

89. In sections reinforced in compression due allowance shall be made for the depth of the compressive reinforcement from the compressed edge of the beam. Compressive reinforcement.

90. The equations in regulations 87 and 88 may be used in conjunction with regulation 61 when the compressive reinforcement is located at or near the centroid of compression in the concrete.

91. In the case of a beam supported at its end by a transverse beam—

(*a*) Longitudinal bars of the supported beam shall be continued across to the further side of the supporting beam, and the ends of the bars shall be hooked. Beams supporting other beams.

(*b*) The whole shear shall be provided for by the tensile resistance of the shear or web reinforcement acting in conjunction with the compressive stresses in the web.

(*c*) Provision shall be made to resist any torsion in the supporting beam, and to resist any negative bending

moments on the supported beam, but the positive bending moments on the supported beam shall be calculated on the assumption of ends being freely supported.

92. At the ends of beams supporting transverse beams the bars shall be so arranged as to give the same margin of safety as required by regulation 91.

Notes on drawings or diagrams.

93. Notes shall be added to the drawings or diagrams of all beams and pillars showing the loads which have been provided for.

## Part IV*

### Pillars and Other Struts.

Strut defined.

94. The term "strut" when used in these regulations shall be deemed to include any compression member at any angle.

Pillar defined.

95. The term "pillar" when used in these regulations shall be deemed to include any pillar, pier, post, column, detached support or any other *vertical* compression member.

96. Pillars shall be designed on the assumption that the concrete and the vertical bars are shortened in length in the same proportion.

Ratio.

97. In calculating the strength of a pillar, the maximum ratio of length to gyration radius or the maximum ratio of length to effective diameter shall be taken.

Length.

98. The length shall be measured between the lateral supports, irrespective of any splayed work in excess of that allowed as part of the beam by regulation 70.

Effective diameter.

99. The effective diameter shall be measured to the outside of the outermost vertical reinforcement and shall be measured in the direction of the lateral supports which determine the length of the pillar.

Reinforcement.

100. All pillars shall be provided with vertical and lateral reinforcement.

* See p. 244.

101. Each pillar with rectilinear laterals (binding) shall have at least four lines of vertical reinforcement throughout its entire length. Binding.

102. Each pillar with curvilinear laterals (binding) shall have at least six lines of vertical reinforcement throughout its entire length.

103. The least diameter of rectilinear laterals (binding) shall not be less than 3/16ths of an inch.

104. The least diameter of curvilinear laterals (binding) shall not be less than 1/8th of an inch.

105. (*a*) The pitch of the laterals (binding) shall not exceed 6/10ths of the effective diameter of a pillar at any part of its length or 16 times the diameter of the least vertical bar.

(*b*) At the ends of a pillar for a length equal to one-and-a-half times the effective diameter the pitch of the laterals (binding) shall not exceed 3/10ths of such effective diameter.

106. The lateral reinforcement (binding) shall be firmly secured at each end.

107. The volume of lateral reinforcement (binding) shall not be less than 0·5 per cent. of the volume of the hooped core.

108. The diameter of vertical bars shall not be less than $\frac{1}{2}$ inch or greater than 2 inches. Vertical bars.

109. The total cross-sectional area of the vertical reinforcement in any pillar shall not be less than 1·0 per cent. of the area of the hooped core.

110. Joints in the vertical reinforcement of pillars shall only be made at or adjacent to a floor level or other point of lateral support. Joints.

111. In all joints in the vertical reinforcement there shall be provided an overlap at least equal to twenty-four times the diameter of the upper bar.

In cases where there may be tension in the pillar the ends of the bars shall be formed as required by regulation 48.

Rectangular pillars.

112. In the case of rectangular pillars in which the ratio between the greater and the lesser diameter exceeds one-and-a-half, the cross-section of the pillar shall be subdivided by cross-ties, and the number of vertical bars shall be such that the distance between the bars along the longer side of the rectangle shall not exceed the distance between the bars along the shorter side of the rectangle.

*Notation for pillars, struts and other compression members.*

Notation.

113.

$A$ = effective *area* of the pillar, i.e. the *area* bounded by the lateral reinforcement (binding) measured to the inside of the hooping.

$A_b$ = cross-sectional *area* of one bar of the *binding* or lateral reinforcement.

$A_v$ = *area* of the *vertical* reinforcement.

$c$ = permissible direct *compressive* stress. (*See regulation* 42.)

$d$ = effective *diameter*. (*See regulation* 99.)

$f$ = *form* factor, depending upon form or type of binding or laterals. (*See table in regulation* 118.)

$g$ = *gyration* radius of the area $[A + (m - 1) A_v]$ about an axis passing through the centroid of the area.

$i$ = *increased* stress permissible in the core of a pillar suitably hooped.

$l$ = actual *length* of the pillar as defined in the regulation numbered 98.

$m$ = *modular* ratio $= \frac{E_s}{E_c}$. (*See regulation* 54.)

$P$ = *permissible* load or *pressure* on pillars with both ends fixed and with a ratio of virtual length to gyration radius not exceeding 45. (*See first column of figures in regulation* 122.)

$p$ = *percentage* of volume of binding or lateral reinforcement with respect to the volume of hooped core in any given length of pillar $= 100\ V_1$.

$p_b$ = *pitch* of the *binding* or lateral reinforcement.

$s$ = *spacing* factor depending upon the spacing or pitch of the binding or laterals. (*See table in regulation* 118.)

$V_1$ = *volume* ratio or *ratio* of the *volume* of binding or lateral reinforcement to the volume of the hooped core in any given length of pillar. (*See regulations* 116 *and* 117.)

$v$ = *virtual* length of the pillar, strut or other compression member for different conditions of the ends. (*See regulations* 122 *and* 124.)

Fixed ends.

114. A pillar, or other strut, shall be deemed to have fixed ends when the ends are sufficiently secured to other parts of the construction having such rigidity as will maintain the axis at the ends in its original position and direction under all loads less than the crippling load.

Formulæ.

115. In a pillar with fixed ends the stress in the concrete in the area bounded by the lateral reinforcement shall not exceed

$c$, in a pillar with the minimum of hooping or binding,

$i$, in a pillar with more than the minimum of hooping or binding, where

$$i = c\,[1 + fsV_1]$$

116. The required ratio of the volume of binding or lateral reinforcement to the volume of the hooped core may be obtained from the equation

$$V_1 = \frac{i - c}{cfs}$$

117. The value of $V_1$ actually provided, in either round or square pillars, may be ascertained by direct measurement or from the equation

$$V_1 = \frac{4A_b}{dp}$$

118.

Table showing the value of $fs$.

| Form of lateral reinforcement or binding | Form factor $f$ | Spacing of laterals throughout the length of the pillar (in terms of diameter of hooped core) | Spacing factor $s$ | Value of $fs$ |
|---|---|---|---|---|
| Helical (curvilinear on plan) | 1·0 | 0·2$d$ or less | 32 | 32 |
| Helical do. | 1·0 | 0·3$d$ | 24 | 24 |
| Helical do. | 1·0 | 0·4$d$ | 16 | 16 |
| Circular hoops ... | 0·75 | 0·2$d$ or less | 32 | 24 |
| Circular hoops ... | 0·75 | 0·3$d$ | 24 | 18 |
| Circular hoops ... | 0·75 | 0·4$d$ | 16 | 12 |
| Rectilinear ... ... | 0·5 | 0·2$d$ or less | 32 | 16 |
| Rectilinear ... ... | 0·5 | 0·3$d$ | 24 | 12 |
| Rectilinear ... ... | 0·5 | 0·4$d$ | 16 | 8 |
| Rectilinear ... ... | 0·5 | 0·5$d$ | 8 | 4 |
| Rectilinear ... ... | 0·5 | 0·6$d$ | 0 | 0 |

119. Notwithstanding any other provision in these regulations the increased stress in the concrete of pillars shall not exceed one-third of the ultimate compressive resistance at four months as given in regulation 159.

120. The working stress in the vertical reinforcement shall not exceed $m$ times the stress in the concrete. (*See regulation* 54.)

121. The permissible load or pressure on pillars with both ends fixed and with ratios not exceeding those in the first column of figures in regulation 122 shall be obtained from either of the equations—

For pillars with the minimum of hooping or binding

$$P = c\,[A + (m - 1)\,A_v]$$

For pillars with more than the minimum of hooping or binding

$$P = i\,[A + (m - 1)\,A_v]$$

122. For pillars the permissible load or pressure shall be obtained from the following table—

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| For the maximum ratio of *virtual* length to *gyration* radius | $\frac{v}{g}=$ | 45 | 54 | 63 | 72 | 81 | 90 |
| For the maximum ratio of *virtual* length to effective *diameter* for square pillars | $\frac{v}{d}=$ | 15 | 18 | 21 | 24 | 27 | 30 |
| For the maximum ratio of *virtual* length to effective *diameter* for round pillars | $\frac{v}{d}=$ | 12 | 15 | 18 | 21 | 24 | 27 |
| Permissible load ... ... | | $P$ | $0{\cdot}8\,P$ | $0{\cdot}6\,P$ | $0{\cdot}4\,P$ | $0{\cdot}2\,P$ | *Nil* |

For other ratios higher than those in the first column of figures the stress shall be proportional to the above.

123. Struts shall be subject to all the rules applying to pillars and the combined stresses at any part shall not exceed the permissible stresses for pillars of like ratios. (*See regulations* 44, 45, 46.) Struts.

124*. For the different conditions of the ends of pillars and other struts mentioned in the following table, the permissible load shall be ascertained by using the corresponding value of the virtual length in such table in conjunction with regulations 121 and 122— As to conditions of ends of struts and pillars.

| Condition of axis at ends | Virtual length, $v$ |
|---|---|
| Both ends fixed in position and direction ... | $v=l$ |
| One end fixed in position and direction and One end fixed in position but not in direction | $v=1{\cdot}4l$ |
| Both ends fixed in position but not in direction | $v=2l$ |
| One end fixed in position and direction and One end not fixed in position and direction | $v=4l$ |

125. In the case of arches or other similar constructions the combined stresses at any part shall not exceed the permissible stresses set out in regulations 42 and 43. Arches. Also see regulations 44, 45, 46.

* See p. 244.

## Part V

### WALLS.

Thickness of walls.

126. Where the dead loads and superimposed loads of, in or upon a building are transmitted to the foundations by a series of reinforced concrete pillars, beams, arches or other constructions designed and constructed in accordance with these regulations, any *external* enclosing *walls of reinforced concrete* between such pillars may be of any thickness not less than *four inches* provided that such enclosing walls are designed and constructed in accordance with these regulations to resist any loads and pressures they may have to carry.

127. In any case where any wall or part of a wall is intended to support vertical loads or resist lateral pressures, it shall be of such thickness as may be necessary to keep the stresses within the limits prescribed by these regulations for the construction of pillars, beams and other members.

128. When portions of the external walls between the reinforced concrete pillars and beams are constructed of brickwork, stonework or plain concrete, such portions of walls shall be of a thickness not less than 8½ inches for the topmost 20 feet of their height and not less than 13 inches for the remainder of their height below such topmost 20 feet.

When such portions are constructed of hollow blocks, the blocks shall conform to the following requirements—

(*a*) The aggregate thickness, including any cavity, shall be at least 8½ inches, measured at right angles to the face of the wall.

(*b*) The sides shall be at least 2 inches thick.

(*c*) The ends and any interior partitions shall be at least 1 inch thick.

(*d*) The clear unsupported span of any part of any side, end, or interior partition shall not be more than four times the least thickness of such part.

(*e*) The aggregate thickness of the material shall be at least 6 inches measured at right angles to the face of the wall.

Provided that a less thickness shall be allowed in any case in which under the London Building Act, 1894, such less thickness is prescribed, and provided that in any case in which an external wall or portion of an external wall is not supported or carried or secured by the reinforced concrete skeleton framed construction within the limit of height and length prescribed by the First Schedule to the London Building Act, 1894, for the purpose of determining the thickness of walls, such external wall or portion of external wall shall be of a thickness not less than that prescribed by such schedule.

Facing.

129. All walls and facing materials shall be incombustible and shall be securely connected to the pillars, beams, floors and other contiguous parts of the reinforced concrete construction.

Pressure on panels.

130. Each panel in any external wall shall be designed to resist safely a horizontal pressure equivalent to a static pressure of at least thirty pounds per square foot assumed to be acting uniformly over the area of one panel from either side. (*See regulations* 21 *and* 22.)

Openings.

131. The aggregate area of openings in external walls constructed in accordance with these regulations shall not exceed in a wall of any storey above the ground storey two-thirds of the whole area of such wall, and the aggregate width of such openings in such a wall shall not exceed three-quarters of the whole length of such wall.

Party and division walls.

132. Party walls and division walls constructed in reinforced concrete in accordance with these regulations shall be of such thickness as may be necessary to comply therewith, but in no part shall such walls be of less thickness than 8 inches.

133. Provided that any such party wall between a building constructed in reinforced concrete and a building of the warehouse class constructed in accordance with the Rules of the London Building Act, 1894, or as a steel-framed building, shall not be in any part less than 13 inches in thickness.

Mortar.

134. All brickwork, stonework and plain concrete shall be executed in Portland cement mortar. The mortar shall be in accordance with the Council's by-laws from time to time in operation.

Pressure on brickwork.

135. The pressure on any brickwork supporting reinforced concrete work shall not exceed the following—

| Brickwork | Tons per square foot |
|---|---|
| Blue brick in cement mortar ... ... ... | 12 |
| Hard brick (including London stock) in cement mortar ... ... ... ... ... ... | 8 |
| Ordinary brick in cement mortar ... ... | 5 |

Ratio of height to thickness.

Such brickwork shall not have a height without proper lateral supports of more than six times its least thickness, but any such brickwork with proper supports may have a height between such lateral supports not more than twelve times the least thickness of such brickwork. Such thickness shall in no case be less than thirteen and a half inches.

## Part VI

### FOUNDATIONS.

Pressure on foundations.

136. The pressure of foundations on the natural ground shall not exceed the following—

| Nature of ground | Tons per square foot |
|---|---|
| Natural bed of soft clay or wet or loose sand ... | 1 |
| Natural bed of ordinary clay or confined sand ... | 2 |
| Natural bed of compact gravel, London blue clay or chalk ... ... ... ... ... | 4 |

137. The pressure on plain concrete in foundations shall not exceed twelve tons per square foot. The plain concrete shall be in cement and its quality shall be at least equal to that required by the Council's by-laws from time to time in operation.

## Part VII

### PROTECTION.

138. The cover shall be measured from the outer surface of the concrete to the outer surface of the metal reinforcement.

End cover.

139. The end cover to be provided beyond the anchored end of a bar coming within the provisions of regulation 48 (*c*) shall not be less than 2 inches and not less than twice the normal diameter of the bar to be covered.

Pillars.

140. The cover of any vertical bar in a pillar shall not be less than $1\frac{1}{2}$ inches and not less than the diameter of such vertical bar.

Beams.

141. The cover of any longitudinal bar in a beam shall not be less than one inch and not less than the diameter of such longitudinal bar.

Slabs.

142. The cover of tensile, compressive, shear or any other reinforcement in slabs shall not be less than one-half inch, and not less than the diameter of the bar to be covered.

Other members.

143. The cover of any reinforcement in other members shall not be less than one inch and not less than the diameter of the bar to be covered.

## Part VIII

### MATERIALS AND TESTING.

*Cement.*

Cement standard.

144. All cement used shall be Portland cement of slow setting quality and shall be in accordance with the British Standard Specification from time to time in operation.

Weight.

145. The quantity of cement shall be determined by weight and ninety pounds shall be deemed to be the equivalent of one cubic foot.

*Sand.*

Sand.

146. The sand shall be clean and gritty. It shall be composed of hard silicious grains, or of materials permitted under regulation 150. It shall be free from clay or any animal, vegetable or bituminous matter.

147. All sand shall pass through a mesh three-sixteenths of an inch square measured in the clear.

148. The sand shall be separated from the coarse material before the materials are measured.

*Coarse Material.*

Coarse material defined.

149. The term "coarse material" means all the ingredients of the concrete except the sand, the cement, and the water.

Permissible materials.

150. The coarse material shall consist of clean Thames or pit-ballast or gravel, hard stone, such as granite, basalt, trap rock, or other hard and equally suitable material. It shall be free from clay or any animal, vegetable, or bituminous matter.

Prohibited materials.

151. The following materials shall not be used with the sand or coarse material in the composition of the concrete under these regulations—

(*a*) Coal residues, including clinkers, cinders, ashes, coke breeze, pan breeze, slag and other similar material.

(*b*) Blast furnace slag, copper slag, forge breeze, dross and other similar material.

(*c*) Sulphates, including plaster of Paris, and other similar material.

(*d*) Limestones, magnesian limestones, marbles and other calcium carbonates.

Washing.

152. Unless quite clean all coarse material shall be thoroughly washed.

Sizes.

153. The coarse material shall be of such a size as will pass through a mesh three-quarters of an inch square measured in the clear and be retained on a mesh three-sixteenths of an inch square measured in the clear.

154. The coarse material shall be varied in size as much as possible between the limits of size allowed for the work, but subject to the provisions of regulation 142 it shall not be larger than such as can pass between the bars forming the reinforcement or between the reinforcement and the centering.

155. The coarse material if of a porous nature shall be thoroughly wetted before being mixed with the other materials. Wetting.

156. The volume of mortar shall be in excess of what would be required to fill completely the interstices and voids of the coarse material. Volumes.

157. The volume of the sand shall not be more than twice the volume of the cement.

158. The volume of coarse material shall not be more than twice the volume of sand.

*Concrete.*

159. The concrete shall be composed of cement, sand and coarse material in one of the following or intermediate proportions and the ultimate compressive resistance shall not be less than that specified for the proportion adopted— Proportions and ultimate resistance.

| Proportion by volume | | | $V$ | *Ultimate* compressive resistance in pounds per sq. in. | | Value of one-fourth of $u_4$ |
|---|---|---|---|---|---|---|
| Cement | Sand | Coarse material | | 1 month after mixing $=u_1$ | or, 4 months after mixing $=u_4$ | |
| 1 | 2 | 4 | 6 | 1600 | 2400 | 600 |
| 1·2 | 2 | 4 | 5 | 1800 | 2600 | 650 |
| 1·5 | 2 | 4 | 4 | 2000 | 2800 | 700 |
| 2 | 2 | 4 | 3 | 2200 | 3000 | 750 |

(*Also see regulations* 42, 145, 148, 156, 158, 162.)

160. For determining the resistance of concrete, tests shall be made on cubes of not less than four inches each way, or cylinders of not less than six inches each way.

Tests. 161. The conditions accompanying the preparation, setting, maturing and actual testing of the cube or other test piece shall, as far as possible, conform to the conditions that would obtain in the actual execution of the reinforced work.

Ultimate resistance. 162. The ultimate compressive resistance of concrete of materials mixed in intermediate proportions may be estimated from the following equation—

$$u_1 = 3600 - 200V, \text{ or}$$

$$u_4 = 3600 - 200V, \text{ where}$$

$u_1$ = *ultimate* compressive resistance at one month, in pounds per square inch.

$u_4$ = *ultimate* compressive resistance at 4 months in pounds per square inch.

$V$ = *Volume* of the sand plus the volume of the coarse material, per volume of cement, each measured separately and including the voids proper to each material.

Mixing. 163. All three materials shall be thoroughly mixed dry in batches, and then again thoroughly mixed in batches after wetting, subject to the requirements of regulation 155.

Salt water shall not be used for the concrete.

Placing. 164. The concrete shall be placed in its final position before initial set has taken place.

In the case of beams, pillars and walls the thickness of the layers of loose concrete shall not exceed three inches before ramming.

Ramming. As soon as possible after mixing, the concrete shall be properly rammed into the moulds in such a manner and under such conditions as will secure a compact mass, without voids and of the greatest possible density for the proportions used.

*Steel.*

Steel standard. 165. All metal reinforcement shall be of steel which shall comply with the British Standard Specification for structural steel for bridges and general building construction from time to time in operation.

166. All metal for reinforcement shall be cleaned of all scale, dust and loose rust, immediately before depositing the concrete. Cleaning.

167. Butt or scarf welding shall not be employed in any tensile reinforcement. No welding.

168. The builder or other person directing the work to be executed shall, for the purpose of due supervision of the construction of the building, furnish the District Surveyor with reasonable proof as to the quality of materials to be used in such construction, and shall make any tests which shall be reasonably necessary. Tests.

## TESTS AND TESTING.

169. If at any time during the construction or within two months after the completion of the reinforced concrete construction it is found necessary to test any part of such construction by reason of any sign of weakness or faulty work appearing in the construction, the builder or other person causing or directing the work to be executed shall make such tests, and, if the tests show the work to be faulty, it shall be reconstructed and reinstated in accordance with these regulations. Tests and testing.

170. The total deflection of beams or slabs freely supported and uniformly loaded and subject to the permissible working stresses shall not exceed 1/600th of the span, when the span is 20 times the effective depth, and shall be in proportion for other ratios of span to depth, and for other conditions of ends and stress and loading. Deflection.

171. The superimposed test load on any floor, roof or other structure shall be not more than one-and-a-half times the superimposed load for which such floor, roof or other structure has been designed. The superimposed test load on any beam, slab or other similar member which has been exposed to frost during the first week of hardening, shall be not less than one-and-a-half times the superimposed load for which such beam, slab or other member has been designed. Test load.

172. Loading tests shall not be made until the expiry of ninety days from the date of laying the concrete.

## Part IX

### FORMWORK OR CENTERING.

Formwork defined.

173. For the purpose of these regulations, the terms "Formwork" or "Centering" shall include all forms, moulds, sheeting, shuttering, planks, poles, posts, shores, struts and strutting, ties, uprights, waling and all other temporary supports to the concrete during the process of setting.

Rigidity.

174. The formwork or centering shall be of such dimensions and so constructed as to remain rigid during the laying, ramming and setting of the concrete.

Strutting.

175. The vertical strutting shall be maintained continuous through the lower storeys to the foundations or to other floors or beams which are sufficiently set to afford the required support without injury to the construction.

Removal.

176. All formwork or centering shall be removed without shock or vibration.

177. Before the formwork or centering under any beam or floor slab is removed the pillars below such beam or floor slab shall be partially stripped so that the pillars may be examined on all sides.

## Part X

### WORKMANSHIP.

Placing.

178. All reinforcement shall be placed and maintained in the position shown on the drawings.

Continuity of work.

179. The concreting in any member shall be carried out as continuously as possible.

Recommencement of work.

180. Where work has to be recommenced on a surface which has hardened, such surface shall be well hacked, swept clean, thoroughly wetted, and covered with mortar composed of equal volumes of cement and sand.

181. Concrete laid during dry weather shall be protected against too rapid drying. During the first week of hardening it shall be kept damp by means of wet sacking or other methods or by watering daily (Sundays and holidays included). Dry weather.

182. Concrete shall not be laid when the temperature is below 4° Centigrade (39° Fahr.), and shall be protected when necessary. Cold weather.

183. Concrete or mortar which has been frozen shall not be used. Frost.

184. The concrete or mortar in any beam, slab, strut, or other similar member which has been exposed to frost during the first week of hardening shall be removed or such members shall be tested as provided for in regulations 169 to 172.

185. No cutting for piping or any other purpose shall be done which would reduce the strength of any part of the structure below the standard required by these regulations. Cutting.

186. Blocks or strips of hard wood, coke breeze or other equally fire-resisting material may be embedded in the concrete and used solely for fixing purposes, provided that— Strips and blocks.

(*a*) They do not reduce the strength of any part of the structure below the standard required by these regulations.

(*b*) The area of such blocks or strips at any given cross-section is not included in the calculated compression area of any beam, slab, pillar, or other constructional member.

187. Soft wood or other equally combustible material shall not be embedded in the concrete. Inlaid materials.

188. Wood or other combustible material may be placed on or over the surface of the concrete, provided that any voids or hollow spaces between the combustible and incombustible materials be filled up with materials of an incombustible nature. Overlaid materials.

## AUTHOR'S NOTES ON L.C.C. REGULATIONS.

62. This regulation is often a great help in design. Unfortunately, design in accordance therewith is unscientific and leads to a progressive reduction in factor of safety as the percentage increases. See Author's research in *Concrete and Constructional Engineering*, May and June 1916.

Shear 64. The whole of the regulations on shear badly need revising.

Part IV 94. The regulations in regard to pillars are unnecessarily complicated. The practical value of binding in raising the working stress is small for all ordinary designs. The much more important factor of moments in columns is ignored and no guide given as to how they may be calculated.

124. These rules, taken in conjunction with Reg. 122, lead to absurdities. Take a column 12″ square, 5′ high, one end free, $v = 4l = 4 \times 60 = 240''$. Then, neglecting the cover (Reg. 99),

$$g \text{ is } \frac{9''}{3{\cdot}46} = 2{\cdot}6''. \quad \frac{v}{g} = \frac{240}{2{\cdot}6} = 92, \text{ Permissible Load Nil.}$$

# INDEX

NOTE.—*Page numbers in italics denote references to L.C.C. regulations.*
*Numbers preceded by S denote number of section of L.C.C. regulations.*
*Numbers preceded by F denote figure numbers.*


Adhesion, *S*49–51 *217*
Alignment diagrams rectangular beams 172–4
Angle of dispersion, *S*18 *209*

Beams, *S*55–93 *218*
Beams, *moments and shears in, neglecting stiffness of columns*
Two spans, uniform loading 2–15
„ central point loading 16–27
„ third point loading 28–39
Many spans, uniform loading 40–57
„ central point loading 58–67
„ third point loading 68–77
Other loadings 78–89
Beams, *moments in, allowing for stiffness of monolithic columns*
One span, uniform loading, *F*73 175–76
„ central point loading, *F*83
„ third point loading, *F*83
Two spans, uniform loading, *F*77 176–84
„ central point loading, *F*84
„ third point loading, *F*85
Many spans 185–203
„ uniform loading, dead load, *F*78
„ uniform loading, live load, *F*82
„ central point loading, dead, *F*86
„ central point loading, live, *F*87
„ third point loading, dead, *F*88
„ third point loading, live, *F*89
Unequal spans, *see* Unequal
Beams, standard T 155–8
Bending moments, *S*29–38 *211–4*
Bending moments, *see* Beams

Centering, *S*173–7 *242*
Columns
Moments in outside, one span 123–36
„ „ more spans 136–8
Moments in interior, two spans
„ „ ends free 139–41
„ „ ends monolithic 141–3
„ „ many spans 144
Columns, *see* Pillars
Columns, standard 147–54
Compressive reinforcement, *S*61–63 *219*
Cover, *see* Protection

Dead load, *S*7 *207*
Dead load 2

Electrical currents, *S*5 *206*

Foundations, *S*136–7 *236*

Hooks, *S*48 *216*

Incombustible materials, *S*4 *206*

Live load allowance for rolling loads 159–62
Live loads, *S*8–17 *207–9*
Live loads 2
Loads 2
Loads, *see* Live, *and* Dead load
Loads reduced in columns, *S*19–20 *209–10*

Materials, *S*144–68 *237–41*
Modular ratio, *S*52–4 *217–8*
Moment, point of maximum 8
Moments, *see* Beams, *or* Columns
Moments, *see* Bending moments

Moments, *see* Resistance moments

Notation x
Notation [L.C.C.], *S*81, 113 *223, 230*
Notices, *S*6 *206*

Parabolic construction 7
Parabolic ordinates 6
Pillars, *S*94–125 *228–33*
Protection, *S*138–43 *237*

Ratio span to depth, *S*25–8 *210*
Reinforced concrete defined, *S*1 *205*
Resistance moments, *S*80–8 *222–7*
Rolling loads 90–3
Rolling loads, live load allowance for 159–62

Scope of L.C.C. regulations, *S*2 *206*
Shear, *S*64–71 *221*
Shear in beams, *see* Beams
Shear resistance 163–71
Slabs, *S*72–9 *221–2*
Standard beams 155–8
Standard columns 147–54
Stresses, *S*42–7 *215–6*
Symbols, *see* Notation

Testing, *S*169–72 *241*
Theorem of three moments 84

Unequal spans,
Two spans, uniform loading 94–100
„ central point loading 101–2
„ other loadings 102–3
Many spans 103–21

Walls, *S*126–35 *234–6*
Weight of concrete, *S*24 *210*
Wind pressure, *S*21–2 *210*
Working stresses, *S*42–7 *215–6*
Workmanship, *S*178–88 *242–3*

CAMBRIDGE: PRINTED BY J. B. PEACE, M.A., AT THE UNIVERSITY PRESS.

UNIVERSITY OF CALIFORNIA LIBRARY
BERKELEY

Return to desk from which borrowed.
This book is DUE on the last date stamped below.

ENGINEERING LIBRARY

AUG 13 1953
JUN 17 1954

LD 21–95m–11,'50(2877s16)476

YC 33000

TA
683
F2
1919
V.2

449483

Engineering
Library

UNIVERSITY OF CALIFORNIA
DEPARTMENT OF CIVIL ENGINEERING
BERKELEY, CALIFORNIA

UNIVERSITY OF CALIFORNIA LIBRARY